13 Oscillations

When a musician strums the guitar, the vibration of the strings produces sounds that the human ear hears as music. When a guitar string is plucked, it moves a certain distance, depending on how hard the guitar player strums. The string returns to its starting point and travels nearly the same distance in the opposite direction. The vibrational energy of the string is dissipated in the form of sound. The amplitude of the vibrations decreases gradually. The volume of the sound fades until the string eventually falls silent.

Topic Notes

PERIODIC AND OSCILLATORY MOTIONS 1

TOPIC 1

REPETITIVE MOTION AND ITS ELEMENTS

When a body is moved slightly from its original position, a force acts to try to return the body to its original position, causing oscillations or vibrations. A periodic motion repeats itself, over and over after a regular interval of time. Oscillatory or vibratory motion is defined as a movement of a body to and fro or back and forth repeatedly about a fixed point over a set period of time. The motion of the pendulum of a wall clock is an example of oscillatory motion. The circular motion is a periodic motion but it is not an oscillatory motion.

Harmonic oscillation can be expressed in terms of a single harmonic function:

i.e., $y = A \sin \square t$ or, $y = A \cos \square t$.

But non-harmonic oscillation cannot be expressed in terms of a single harmonic function. It is a combination of two or more than two harmonic functions.

i.e., $y = A \sin \omega t + B \sin 2\omega t$.

Time Period and Displacement

Time taken to complete one vibration is called a periodic time of Simple harmonic motion.

In the equation of Simple harmonic motion,

$$x = (\omega t + \phi).$$

if time t is increased by $\frac{2\pi}{\omega}$, the displacement becomes,

$$x = \left[\omega\left(t + \frac{2\pi}{\omega}\right) + \phi\right]$$

$$x = (\omega t + 2\pi + \phi)$$

$$x = (\omega t + \phi).$$

It means that the displacement repeats itself after a time $\frac{2\pi}{\omega}$. Therefore, $\frac{2\pi}{\omega}$ is called time period T of the motion is,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Time period is independent of the motion.

Frequency (ν)

Number of oscillations that occurred in a second *i.e.*,

$$\nu = \frac{1}{T}.$$

The unit of ν is s^{-1}.

After the discoverer of radio waves, Heinrich Rudolph Hertz (1857–1894), a special name has been given to the unit of frequency. It is called hertz (abbreviated as Hz).

Thus, 1 Hertz = 1 Hz = 1 oscillation per second = $1 s^{-1}$.

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Example 1.1: Which of the following examples represents periodic motion?

(A) A swimmer completing one (return) trip from one bank of a river to another and back.

(B) A freely suspended bar magnet displaced from its North-South direction and released.

(C) A hydrogen molecule rotating about its centre of mass.

(D) An arrow released from a bow. [NCERT]

Ans. (A) It is not a periodic motion, as he does not take a definite time to repeat his motion

(B) It is a periodic motion, as it oscillates in a North-South direction.

(C) It is a periodic motion.

(D) It is not a periodic motion.

Example 1.2: Glass window panels are sometimes broken by an explosion several miles away. Explain why?

Ans. When an explosion takes place at a faraway place, the disturbance reaching the glass panels sets, the glass panels into vibration and if the frequency of vibration is equal to the natural frequency of the window panel, then resonant vibration takes place and hence window panels break.

TOPIC 2

SIMPLE HARMONIC MOTION

Simple harmonic motion is a type of oscillatory motion in which:

(1) The particle moves in one dimension.

(2) The particle moves to and fro about a fixed mean position (Where, $F_{net} = 0$).

(3) The net force on the particle is always directed towards the mean position.

(4) The magnitude of the net force is always proportional to the particle's displacement from the mean position at that instant.

When a particle always moves towards the mean position, its acceleration is directly proportional to its displacement from the mean position but is directed away from displacement, it is said to execute SHM.

i.e., $a \propto x$

For simple harmonic motion, the displacement $x(t)$ of the particle from a certain chosen origin or mean position is found to vary with time(t) as,

$$x(t) = A \cos (\omega t + \phi)$$

Where, $x(t)$ = Displacement,

A = Amplitude

ωt = Angular velocity

ϕ = Phase control

This equation of motion represents Simple harmonic motion.

Important

➥ *The motion of molecules of a solid, the vibration of the air columns and the vibration of the string of musical instruments are either simple harmonic or superposition of simple harmonic motion.*

Amplitude

The maximum displacement on either side of the mean position of a particle executing S.H.M. is defined as its amplitude. The particle's amplitude is denoted by A.

In the equation, $x = A \sin (\omega t \pm \phi)$, the value of $\sin \omega t$ varies from + 1 to − 1. So, the displacement varies from + A to − A. The maximum value of displacement of particle x = A, which is the amplitude of the motion.

Phase

The quantity $(\omega t + \phi)$ is called a phase of the motion. Constant ϕ is the initial phase, *i.e.*, the value of phase at $t = 0$. The phase of a vibrating particle at any instant defines the state of the particle regarding its position and direction of motion at that instant.

Phase Constant

If the initial position of the particle is not the mean position of the maximum displacement then a phase constant ϕ is added to the displacement equation.

$$x = A \cos (\omega t \pm \phi)$$

or $$x = A \sin (\omega t \pm \phi)$$

Displacement

In the case of SHM, displacement is represented by $x = A \sin \omega t$ or $x = A \cos \omega t$

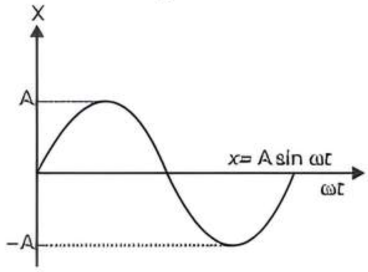
where, 'A' is the amplitude or maximum displacement from the mean position and ω is angular frequency.

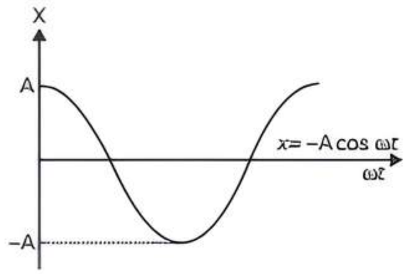
Mathematically, we can represent displacement as a function of time, as $x = ut + \frac{1}{2} gt^2$.

In the case of periodic motion, the displacement is a function of time, such that it is periodic as in

$$x = f(t) = A \sin \omega t$$

or $$x = f(x) = A \cos \omega t.$$





Important

➥ *The direction of displacement is always away from the mean position whether the particle is moving from or coming towards the mean position.*

Caution

➥ *Students should know that harmonic oscillations are oscillations which can be expressed in terms of a single harmonic function.*

Example 1.3: **The motion of a particle executing SHM is described by the displacement function,**

$$x(t) = A\cos(\omega t + \phi).$$

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is ω cm/s. What are its amplitude and initial phase angle? The angular frequency of the particle is πs^{-1}. If instead of the cosine function, we choose the sine function to describe the SHM:

$$x = B\sin(\omega t + \alpha),$$

What are the amplitude and initial phase of the particle with the above initial conditions? [NCERT]

Ans. Here,
$t = 0,$
$x = 1$ cm,
$v = \omega$ cm/s,
$\phi = ?,$

$$\omega = \frac{\pi}{x}$$

$$x = A\cos(\omega t + \phi)$$
$$1 = A\cos(\pi \times 0 + \phi)$$
$$= A\cos\phi \quad \text{...(i)}$$

Velocity,
$$v = \frac{dx}{dt}$$
$$= -A\omega\sin(\omega t + \phi)$$
$$= -A\omega\sin(\pi \times 0 + \phi),$$
$$1 = -A\sin\phi$$

or $A\sin\phi = -1$ ...(ii)

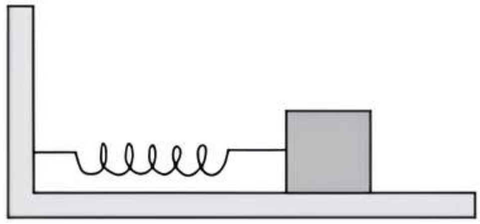
Squaring and adding equations (i) and (ii),
$$A^2(\cos^2\phi + \sin^2\phi) = 1 + 1 = 2$$
$$A^2 = 2$$
or $A = \sqrt{2}$ cm

Dividing equation (ii) by (i),
$$\tan\phi = -1$$
or $\phi = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$

For $x = B\sin(\omega t + \alpha)$

Solving similarly, we get,
$$B = \sqrt{2} \text{ cm}$$
and phase angle,
$$\alpha = \frac{\pi}{4} \text{ or } \frac{5\pi}{4}$$

Example 1.4: A spring with a spring constant 1200 Nm^{-1} is mounted on a horizontal table as shown in Fig. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released. Determine (A) the frequency of oscillations, (B) the maximum acceleration of the mass and (C) the maximum speed of the mass.



[NCERT]

Ans. Given,
$k = 1200$ N/m;
$m = 3$ kg;
$x = 2$cm $= 0.02$ m

(A) Frequency,
$$\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
$$= \frac{1}{2 \times 3.14}\sqrt{\frac{1200}{3}}$$
$$= 3.2 \text{ s}^{-1}$$

(B) Acceleration, $\omega^2 y = \frac{k}{m}y$

Maximum Acceleration, $\frac{kx}{m}$
$$= \frac{1200 \times 0.02}{3} = 8 \text{ m/s}^2$$

(C) Maximum speed, $x\omega$
$$= x\sqrt{\frac{k}{m}}$$
$$= 0.02 \times \sqrt{\frac{1200}{3}} = 0.4 \text{ m/s}$$

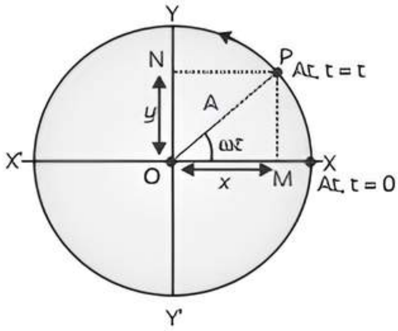
TOPIC 3

SIMPLE HARMONIC MOTION AND UNIFORM CIRCULAR MOTION

If a particle moves at a constant speed around the circumference of a circle, the straight line motion of the particle's perpendicular foot on the diameter of the circle is known as Simple harmonic motion.

SHM based on Circular Motion

Let's draw a circle with radius A, equal to the particle's amplitude when performing SHM. Assume a particle moves in a circle with constant angular velocity ω. SHM can be seen when the particle position is perpendicular to the vertical and horizontal diameters. After time t, the radius vector rotates by ωt.



As a result, if the projection of P is taken on the Y-axis. Then from the figure.

$$y = A \sin \omega t$$

$$y = A \sin \frac{2\pi}{T} t$$

$$y = A \sin 2\pi n t$$

(1) $y = A \sin \omega t$, when the time is noted from the instant, the vibrating particle is at the mean position.

(2) $y = A \cos \omega t$ when time is measured from the instant, the vibrating particle is at its most extreme position.

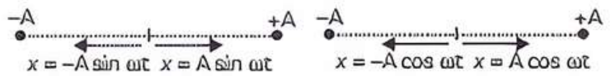
(3) $y = A \sin (\omega t \pm \phi)$ when the vibrating particle is ϕ, phase leading or lagging from the mean position.

If the projection of P is taken on *X-axis* then equations of S.H.M. can be given as:

$$x = A \cos(\omega t \pm \phi)$$

$$x = A \cos\left(\frac{2\pi}{T} t \pm \phi\right)$$

$$x = A \cos(2\pi n t \pm \phi)$$

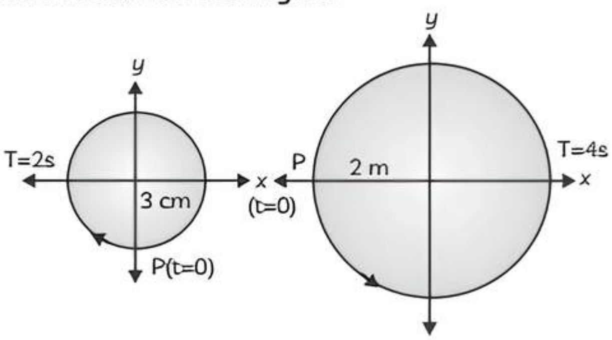


Direction of displacement is always taken away from the equilibrium position, a particle either is moving away from or is coming towards the equilibrium position.

⚠ Caution

➥ *Students should know that in linear SHM, the length of the SHM path is 2A, the total work done and displacement in one complete oscillation is zero but the total travelled length is 4A.*

Example 1.5: **Following figures correspond to two circular motions. The radius of the circle, the period of revolution, the initial position and the sense of revolution (*i.e.*, clockwise or anticlockwise) are indicated on each figure.**

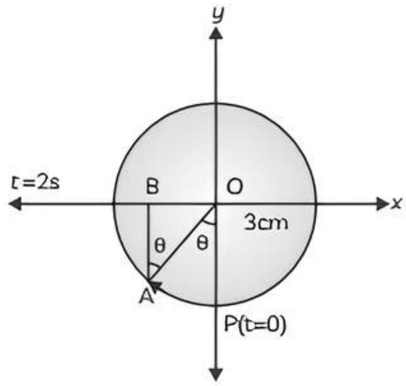


(A) (B)

Obtain the corresponding simple harmonic motions of the *x*-projection of the radius vector of the revolving particle P, in each case. [NCERT]

Ans. (A) Let the radius vector OA is projected at point B on the diameter of the circle and make an angle θ with initial position P.

Thus, $\angle POA = \theta = \angle BAO$



In ΔOBA,

$$OB = OA \sin \theta$$

Or $$-x = 3 \sin \theta = 3 \sin \omega t$$

Or $$x = -3 \sin \frac{2\pi}{2} t$$

$$\left[\omega = \frac{2\pi}{t} \text{ and } t = 2s\right]$$

$$x = -3 \sin \pi t$$

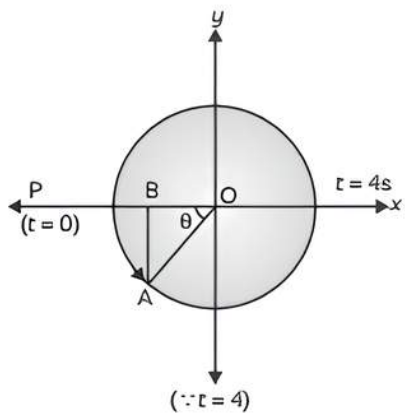
(B) Let the radius vector OA be projected at point B on the diameter of the circle and make an angle θ with the initial position.

Thus $\angle AOB = \theta$

In ΔOBA,

$$\frac{OB}{OA} = \cos \theta$$

$$OB = OA \cos \theta$$



Or $$-x = 2 \cos \frac{2\pi}{4} t$$

$$x = -2 \cos \frac{\pi}{2} t$$

Example 1.6: Case Based:

Simple harmonic motion is the simplest form of oscillation. A particular type of periodic motion in which a particle moves to and fro repeatedly about a mean position under the influence of a restoring force is termed as simple harmonic motion (S.H.M).

A body is undergoing simple harmonic motion if it has an acceleration, which is directed towards a fixed point and proportional to the displacement of the body from that point.

Acceleration, $a \propto -x$

$\Rightarrow$ $a = -kx$

or $\frac{d^2x}{dt^2} = -kx$

Where, x = displacement at any instant t.

(A) Assertion (A): Simple harmonic motion is the projection of uniform circular motion.

Reason (R): Simple harmonic motion is a uniform motion.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

(B) Frequency of oscillation of a body is 6 Hz when force F_1 is applied and 8 Hz when F_2 is applied. If both forces F_1 and F_2 are applied together then, the frequency of oscillation is:

(a) 14 Hz (b) 2 Hz

(c) 10 Hz (d) $10\sqrt{2}$ Hz

(C) A particle executing simple harmonic motion has a period of 6 s. The time taken by the particle to move from the mean position to half the amplitude, starting from the mean position is:

(a) $\frac{1}{4}$ s (b) $\frac{3}{4}$ s

(c) $\frac{1}{2}$ s (d) $\frac{3}{2}$ s

(D) A 10 kg strap is attached to a spring (spring constant 600 N/m), and it slides over a horizontal rod without friction. The strap is moved 20 cm away from its equilibrium position and then released. What is the oscillator's speed?

(E) Name two practical examples of Simple Harmonic Motion.

Ans. (A) *(c) A is true but R is false.*

Explanation: Simple harmonic motion, $v = \omega\sqrt{a^2 - y^2}$ as y changes, velocity v will also change. So simple harmonic motion is not uniform motion. But simple harmonic motion may be defined as the projection of uniform circular motion along one of the diameter of the circle.

(B) *(c) 10 Hz*

Explanation: According to the question,

$$F_1 = -k_1x \text{ and } F_2 = -k_2x$$

$$v_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} = 6 \text{ Hz}$$

$$v_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} = 8 \text{ Hz}$$

Now $F = F_1 + F_2$

$= -(k_1 + k_2)x$

Therefore,

$$n = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{4\pi^2n_1^2m + 4\pi^2n_2^2m}{m}}$$

$$= \sqrt{n_1^2 + n_2^2}$$

$$= \sqrt{8^2 + 6^2}$$

$$= 10 \text{ Hz}$$

(C) *(c)* $\frac{1}{2}$ *s*

Explanation: Given, T = 6 s

When the particle starts from the mean position equation of SHM is :

$$x = A \sin \omega t$$

Let particle reach $x = \frac{A}{2}$ distance in t sec

$$\frac{A}{2} = A \sin \omega t$$

$$\frac{1}{2} = \sin \omega t$$

$$\omega t = \frac{\pi}{2}$$

$$t = \frac{T}{12} \qquad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$t = \frac{6}{12}$$

$$= \frac{1}{2} \text{ s}$$

Therefore $\frac{1}{2}$ seconds is the time taken.

(D) Angular frequency of spring - block system is given by:

$$\omega = \sqrt{\frac{k}{m}}$$

Maximum speed in oscillation.

$$v_{max} = A\,\omega = \sqrt{\frac{k}{m}}$$

Here, $A = 20$ cm $= 0.2$ m.

$k = 600$ Nm.

$m = 10$ kg.

$v_{max} = ?$

$$v_{max} = 0.2\sqrt{\frac{600}{10}}$$

$$= 0.2\sqrt{60} \text{ m/s}$$

(E) (1) Atoms vibrating in a crystal lattice.

(2) Motion of a helical spring.

TOPIC 4

VELOCITY AND ACCELERATION IN SIMPLE HARMONIC MOTION

The velocity of a particle is given by.

$$v = \frac{dx}{dt}$$

In SHM, displacement of the particle is given by.

$$x = A\sin(\omega t \pm \phi)$$

Or $$\frac{dx}{dt} = v = A\cos(\omega t \pm \phi)$$

$$v = A\sqrt{1-\sin^2(\omega t \pm \phi)}$$

$$v = A\omega\sqrt{1-\frac{x^2}{A^2}}$$

$$v = \omega\sqrt{A^2 - x^2}$$

$v_{max} = A\omega$, at the mean position.

$v_{min} = 0$, at the extreme position.

Acceleration of a particle is given by.

$$a = \frac{dv}{dt}$$

In SHM, the velocity of a particle is given by.

$$v = -\omega\sin(\omega t + \phi).$$

on differentiating,

$$a = \frac{d}{dt}(-\omega A\sin(\omega t + \phi)).$$

$$= \omega^2 x$$

$$A = 0 \text{ at } x = A$$

$$a_{max} = -\omega^2 A.$$

$$a_{min} = 0 \text{ in the mean position}$$

or at $$x = \pm A$$

⚠ Caution

➥ *Students should know that in SHM, the velocity and acceleration vary simply harmonically with the same frequency as displacement.*

➥ *Acceleration is ahead of displacement by phase angle π radian i.e. opposite to displacement.*

Important

➥ *Velocity is always ahead of displacement by phase angle $\frac{\pi}{2}$ radian and acceleration leads the velocity by phase angle $\frac{\pi}{2}$ radian.*

Example 1.7: Two identical pendulums are executing (approximate) SHMs with amplitudes *a* and *na* respectively. Calculate the ratio of their energies of oscillation.

Ans. According to the question, we know that energy for two identical pendulums.

$$E_1 = \frac{1}{2}ka^2.$$

$$E_2 = \frac{1}{2}k(na)^2$$

$$\frac{E_1}{E_2} = \frac{a^2}{(na)^2}$$

$$\frac{E_1}{E_2} = \frac{1}{n^2}$$

Example 1.8: How is the time period of the pendulum affected when the pendulum is taken to hills or in mines?

Ans. On the top of a mountain or below the earth, the values of *g* are less than that on the surface of the Earth. With a decrease in the value of *g*, the time period of the simple pendulum increases and accordingly, the pendulum loses time.

TOPIC 5

FORCE LAW FOR SIMPLE HARMONIC MOTION

According to the force law for simple harmonic motions, "When a particle is subjected to a force that is proportional to the particle's displacement and directed towards the mean position, the particle executes SHM and the system is referred to as a linear harmonic oscillator".

As $F = ma$

$= -kx$

$$-\frac{x}{a} = \frac{m}{k}$$

$$ma = -m\omega^2 x$$

$$= -m\frac{4\pi^2}{T^2}x \left[\because \omega = \frac{2\pi}{T}\right]$$

$$T = 2\pi\sqrt{\frac{x}{a}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Time Period, $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

or $T = 2\pi\sqrt{\frac{\text{inertial factor}}{\text{spring factor}}}$

Frequency, $\nu = \frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}}$

or $\nu = \frac{1}{2\pi}\sqrt{\frac{\text{spring factor}}{\text{inertial factor}}}$

Important

➥ *In simple harmonic motion, the acceleration of the system and therefore, the net force is proportional to the displacement and acts in the opposite direction of the displacement.*

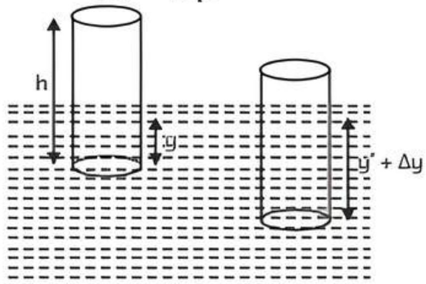
Example 1.9: A cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_l. The cork is depressed slightly and then released. Show that, the cork oscillates up and down harmonically with a period $T = 2\pi\frac{h\rho}{\rho_l g}$.

where ρ is the density of the cork. (Ignore damping due to the viscosity of the liquid).

Ans. Let initially in equilibrium y height of the cylinder is inside the liquid. Then,

Weight of the cylinder = upthrust due to liquid displaced

$$Ah\rho g = Ay\rho_l g$$



When the cylindrical cork is depressed slightly by Δy and released, a restoring force, equal to additional up thrust, acts on it.

The restoring force,

$$F = A(y + \Delta y)\rho_l g - Ay\rho_l g$$

$$= Ag\Delta y$$

Acceleration, $a = \frac{F}{m} = \frac{A\rho_l g\Delta y}{Ah\rho} = \frac{\rho_l g}{h\rho}\cdot \Delta y$

and the acceleration is directed in a direction opposite to Δy.

Obviously, as a $(\alpha - \Delta y)$, the motion of the cork cylinder is SHM, whose time period is given by,

Time Period, $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

$$T = 2\pi\sqrt{\frac{\Delta y}{a}}$$

$$T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Two particles oscillate with the same frequency and amplitudes together across two close parallel straight lines side by side. When their displacement is half of the amplitude, they pass each other, moving in opposite directions. The average positions

of the two particles lie in a straight line perpendicular to their paths. The difference in phase is:

(a) $\frac{\pi}{6}$ (b) 0

(c) $\frac{2\pi}{3}$ (d) π

Ans. *(c)* $\frac{2\pi}{3}$

Explanation:

$$x_1 = A \sin(\omega t + \phi_1)$$

$$x_1 = \frac{A}{2}$$

$$\frac{A}{2} = A \sin(\omega t + \phi_1)$$

$$\frac{1}{2} = \sin(\omega t + \phi_1)$$

$$\omega t + \phi_1 = \frac{\pi}{6}$$

$$x_2 = A \sin(\omega t + \phi_2)$$

$$x_2 = \frac{A}{2}$$

$$\frac{A}{2} = A \sin(\omega t + \phi_2)$$

$$\left(\pi - \frac{\pi}{6}\right) = \omega t + \phi_2$$

(As particle 2 is moving towards a mean position at $\frac{A}{2}$ displacement)

$$\text{Phase difference} = \phi_2 - \phi_1$$

$$= \left(\frac{\pi - \pi}{6}\right) - \omega t - \left[\frac{\pi}{6} - \omega t\right]$$

$$= \pi - 2 \times \frac{\pi}{6}$$

$$= \frac{2\pi}{3}$$

Caution

➥ *Students should know that every oscillatory motion is not simple harmonic motion while every simple harmonic motion is oscillatory. The oscillatory motions, for which the restoring force is linear, are simple harmonic, or the constrained periodic motion with linear restoring force is simple harmonic motion.*

2. At 6 seconds, the displacement from the mean position of a particle in SHM is $\frac{1}{\sqrt{2}}$ of the amplitude. Its duration will be:

(a) 48 sec (b) $\frac{6}{\sqrt{3}}$ sec

(c) 9 sec (d) 0

Ans. *(a) 48 sec*

Explanation: We know that equation is,

$$y = A \sin(\omega t)$$

at $t = 6$ sec,

$$y = \frac{1}{\sqrt{2}}$$

So $$\sin(\omega \times 6) = \frac{1}{\sqrt{2}}$$

So, $$6\omega = \frac{\pi}{4}$$

$$\omega = \frac{\pi}{24}$$

$$T = \frac{2\pi}{\omega} = 48 \text{ sec}$$

3. If a simple harmonic oscillator has a displacement of 0.03 m and an acceleration of 6.0 m/s^2, at any time. The oscillator's angular frequency is equal to:

(a) 10 rad/sec (b) 20 rad/sec

(c) 14.14 rad/sec (d) 20.14 rad/sec

Ans. *(c) 14.14 rad/sec*

Explanation: When a particle undergoes SHM, its acceleration is given by,

$$\alpha = \omega^2 x$$

Given, $\alpha = 6,$

$$x = 0.03.$$

Using these values,

$$\omega = \sqrt{\frac{\alpha}{x}} = \sqrt{\frac{6}{0.03}}$$

$$= 14.14 \text{ sec}$$

Related Theory

➥ *If the frequencies of two SHMs are exactly equal, the resultant path remains steady. But if there is a slight difference between frequencies, the relative phase of the two components of motion changes.*

4. A 3 g body is executing S.H.M. around a fixed point O. It has a maximum velocity of 150 cm/s and amplitude of 5 cm. At a distance (in cm), its velocity will be 90 cm/s is:

(a) $4\sqrt{3}$ cm (b) 4 cm

(c) 14.14 cm (d) 20 cm

Ans. *(b) 4 cm*

Explanation: Angular velocity,

$$v_{max} = 150 \text{ cm/s}$$

$$A = 5 \text{ cm}$$

$$150 = 5\omega$$
$$\omega = 30 \text{ rad/s}$$

As $v = \omega\sqrt{A^2 - y^2}$.

$$v = 90 \text{ cm/s}$$
$$90 = 30\sqrt{5^2 - y^2}$$
$$y^2 = 25 - 9$$
$$= 16$$

Or $y = 4$ cm

5. **The equation, $x = 5.0 \cos(2\pi t + \pi)$, describes how a body oscillates with SHM. Its displacement, speed and acceleration are as follows at time $t = 1.5$s:**

(a) $0, -10\pi, +20\pi^2$ **(b)** $5, 0, -20\pi^2$
(c) $2.5, +20\pi, 0$ **(d)** $-5.0, +5\pi, -10\pi^2$

Ans. *(b) $5, 0, -20\pi^2$*

Explanation: $x = 5\cos(2\pi t + \pi)$

$t = 1.5$ sec

Displacement,

$$x = 5\cos(2\pi \times 1.5 + \pi)$$
$$= 5\cos(4\pi)$$
$$x = 5 \text{ m}$$

Velocity, $v = \dfrac{dx}{dt}$

$$v = -10\pi \sin(2\pi t + \pi)$$
$$v = -10\pi \sin(2\pi \times 1.5 + \pi)$$
$$v = -10\pi \sin(4\pi)$$
$$v = 0 \text{ m/s}$$

Acceleration, $a = \dfrac{dv}{dt}$

$$a = -20\pi^2 \cos(2\pi t + \pi)$$
$$a = -20\pi^2 \cos(2\pi \times 1.5 + \pi)$$
$$a = -20\pi^2 \cos(4\pi)$$
$$a = -20\pi^2$$

⚠ Caution

➥ *Students should know that displacement, speed and acceleration differs in polarity. Always be careful while defining signs to their respective values as it might alter the answer.*

6. **The maximum velocity of a particle, executing simple harmonic motion with an amplitude 12 mm, is 6.2 m/s. The period of oscillation is:**

(a) 10 s **(b)** 12.10 s
(c) 12.15 s **(d)** 13 s

Ans. *(c) 12.15 s*

Explanation: For a SHM,

$$x = A\sin(\omega t + \phi)$$

Velocity, $v = \dfrac{dx}{dt}$

$$= A\omega\cos(\omega t + \phi)$$

So, $v_{max} = A\omega$

or $6.2 = 7 \times 10^{-3}\left(\dfrac{2\pi}{T}\right)$

or $T = 12.15$ s

7. **The displacement of a particle is represented by the equation. $y = \sin^3 \omega t$. The motion is:**

(a) non-periodic.
(b) periodic but not simple harmonic.
(c) simple harmonic with period $\dfrac{2\pi}{\omega}$.
(d) simple harmonic with period $\dfrac{\pi}{\omega}$.

[NCERT Exemplar]

Ans. *(b) periodic but not simple harmonic.*

Explanation: A motion will be harmonic if $A \propto$ displacement and simple harmonic motion is always periodic but all periodic are not harmonic.

$$y = \sin^3 \omega t$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$
$$4\sin^3\theta = 3\sin\theta - \sin 3\theta$$
$$\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$
$$y = \frac{3\sin\omega t - \sin 3\omega t}{4}$$
$$v = \frac{dy}{dt}$$
$$v = \frac{1}{4}[3\omega\cos\omega t - 3\omega\cos 3\omega t]$$
$$v = \frac{3\omega}{4}[\cos\omega t - \cos 3\omega t]$$
$$a = \frac{3\omega}{4}[-\omega\sin\omega t + 3\omega\sin 3\omega t]$$

a is not directly proportional to y. So, motion is not harmonic.

$$y(t) = \sin^3 \omega t$$
$$y(t+T) = \sin^3[\omega(t+T)]$$
$$= \sin^3\left[\frac{2\pi}{T}T + \omega t\right]$$
$$= \sin^3(2\pi + \omega t)$$
$$= \sin^3 \omega t$$
$$y(t+T) = y(t)$$

So, the function is periodic.

Hence, the given function of motion is periodic but not harmonic.

⚠ Caution

➥ *Students should know that in SHM the velocity and acceleration vary simply harmonically with the* *same frequency as displacement. Acceleration is ahead of displacement by phase angle π radian i.e. opposite to displacement. Velocity is always ahead of displacement by phase angle $\frac{\pi}{2}$ radian and acceleration leads the velocity by phase angle $\frac{\pi}{2}$ radian.*

8. The periodic time of a body executing simple harmonic motion is 3 s. After how many interval from $t = 0$, its displacement will be half of its amplitude?

(a) $\frac{1}{8}$ s (b) $\frac{1}{6}$ s

(c) $\frac{1}{4}$ s (d) $\frac{1}{3}$ s

[Delhi Gov. QB 2022]

Ans. *(c)* $\frac{1}{4}$ s

Given that, time period T = 3 seconds.

The equation for SHM is given by,

$y = A \sin \omega t$ and $\omega = 2\pi f = \frac{2\pi}{T}$

Let after time $t = t_1$ the displacement is half of its amplitude.

$$\Rightarrow \quad \frac{A}{2} = A \sin\left(\frac{2\pi}{3}t\right)$$

$$\Rightarrow \quad \sin\left(\frac{\pi}{6}\right) = \sin\left(\frac{2\pi t}{3}\right)$$

$$\Rightarrow \quad \frac{\pi}{6} = \frac{2\pi t}{3}$$

or, $$t = \frac{1}{4} \text{ sec}$$

Assertion-Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

9. Assertion (A): A simple pendulum is mounted on a truck which moves with constant velocity. The time period of the pendulum will increase.

Reason (R): The effective length of the pendulum will decrease.

Ans. *(d) A is false and R is also false.*

Explanation: The time period will be the same. Time period is the time taken by a pendulum to complete one oscillation. If it is inside a truck, as the train undergoes horizontal motion, it will not have any impact on the motion of the pendulum. Hence, the time period and frequency will be the same. Only velocity with respect to ground will differ.

10. Assertion (A): When a particle is at an extreme position performing SHM, its momentum is equal to zero.

Reason (R): At an extreme position, the velocity of a particle performing SHM is equal to zero.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: In SHM, kinetic energy is maximum at the mean position and zero at the extreme positions, while potential energy is zero at the mean position and maximum at the extreme positions. At the equilibrium position, the velocity is at its maximum and the acceleration (*a*) has fallen to zero. Simple harmonic motion is characterised by this changing acceleration that always is directed toward the equilibrium position and is proportional to the displacement from the equilibrium position.

11. Assertion (A): When a person sitting on a glide, stands up, the swing's periodic time increases.

Reason (R): The effective length of the swing will decrease in a girl's standing position.

Ans. *(d) A is false and R is also false.*

Explanation: A girl is sitting on a swing when she stands up, the periodic time of the swing will increase. Now, when the girl is sitting on the swing and swinging, the centre of gravity of the girl is close to the ground meaning at a higher distance from the point of suspension. When the girl stands up the centre of gravity shifts to a higher level from the ground, hence the effective length of the swing decreases. Now, from the formula of time period, we know,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

So, in this problem, $l_1 > l_2$.

Hence, the time period of the swing will be, $T_1 > T_2$.

12. **Assertion (A):** All oscillatory motions are necessarily periodic motions but all periodic motions are not oscillatory.

Reason (R): Simple pendulum is an example of oscillatory motion.

Ans. (b) *Both A and R are true and R is not the correct explanation of A.*

Explanation: All oscillatory motions are necessarily periodic motions, but air periodic motions are not oscillatory.

Simple pendulum is an example of oscillatory motion.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

13. *Bungee jumping is an example of simple harmonic motion. The long elastic rubber is tied to the ankle of the person who then jumps off from the bridge or a certain height. The jumper is oscillating down and up and undergoes Simple Harmonic Motion due to the elasticity of the bungee cord, albeit at decreasing altitude. When the elasticity comes to rest, the jumper starts swinging like a pendulum till he is pulled back up.*

(A) **For the problem of the block acted on by spring described by the equation, $a = -\frac{k}{m}x$, find the position, where the acceleration is smallest and also shows that no matter which way the block is moving, it will eventually turn around and move the other way.**

(B) **What is the minimum condition for a system to execute S.H.M?**

(C) **Mention the types of oscillatory motion.**

Ans. (A) From equation $a = -\frac{k}{m}x$, a vanishes, when $x = 0$ that is the unstretched position of the spring and if the block is moving to the right beyond the unstretched position, it will experience an acceleration that will slow it down. Furthermore, the strength of that acceleration will increase as x increases, confirming that it will eventually be strong enough to reverse the direction of the velocity. The same reasoning applies when the block is moving to the left past the unstretched position. In that case, the acceleration is to the right and the block will again reverse its direction of motion.

(B) The minimum condition for a body to possess S.H.M. is that it must have elasticity and inertia.

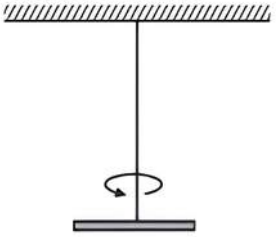
(C) There are mainly two types of oscillatory motion i.e., linear and circular oscillatory motion.

Linear Oscillatory Motion: The object goes left and right or up and down in linear oscillatory motion. For example the vibration of strings of musical instruments.

Circular Oscillatory Motion: In Circular Oscillatory Motion, the object moves left to right in a circular motion. For example, the motion of a solid sphere in a half, hollow sphere.

14. *A torsion pendulum is analogous to a mass-spring oscillator. Instead of a mass at the end of a helical spring, which oscillates back and forth along a straight line, however, it has a mass at the end of a torsion wire, which rotates back and forth. To set the mass spring in motion, you displace the mass from its equilibrium position by moving it in a straight line and then releasing it. The helical spring (or gravity, depending on whether or not the system is oriented vertically, and in which direction you displace the mass) exerts a (linear) force to restore the mass to its equilibrium position. To set the torsion pendulum oscillating, you turn the mass (rotate it about its center), and then release it. To do this, you must exert torque about the bottom of the torsion wire. The torsion wire, in turn, exerts a restoring torque to bring the mass back to its original position.*

(A) If a bar, at the end of a torsion wire, is twisted through some angle and then released, what is the relationship between the angular acceleration α and angular displacement θ?



(a) $\alpha = -\frac{\delta}{I}\theta$ (b) $\alpha = \frac{\delta}{I}\theta$

(c) $\alpha = \delta\theta$ (d) $\alpha = I\theta$

(B) A body executes simple harmonic motion of amplitude 1 cm and frequency 12 cycles per second, then at 0.5 cm displacement, velocity will be:

(a) 60.2 cm/sec (b) 65.3 cm/sec
(c) 83.2 cm/sec (d) 77.3 cm/sec

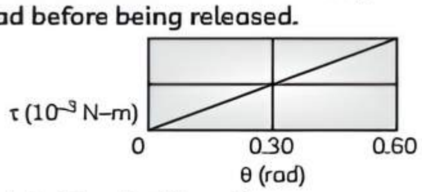
(C) Assume for the above-mentioned figure in (A) the torsion constant is, δ = 1000 N *m*/rad and the moment of inertia, I = 0.500 kg m^2. The bar is rotated through an angle of π rad and released from rest, then its period and amplitude of motion is:

(a) 0.140 s and 3.14 rad
(b) 1.042 s and 2.14 rad
(c) 1.042 s and 3.14 rad
(d) 0.140 s and 2.14 rad

(D) The time period of oscillations of a torsional pendulum, if the wire's torsional constant is, K = 15 π^2 J/rad. The rigid body's moment of inertia is 5 $kgm.^2$ about the rotational axis is:

(a) 2 s (b) 1.04 s
(c) 1.154 s (d) 3 s

(E) Torsion pendulums are made of a metal disc with a soldered wire running through the centre. The wire is clamped vertically and pulled tautly. The magnitude *t* of the torque required to rotate the disc about its centre (and thus twist the wire) versus the rotation angle is shown in the figure. The vertical axis scale is defined as, $\tau^2 = 6 \times 10^{-3}$ Nm. The disc is rotated to a value of, θ = 0.300 rad before being released.



(a) 8.11×10^{-5} kgm^2
(b) 7.11×10^{-5} kgm^2
(c) 3.21×10^{-5} kgm^2
(d) 8.0×10^{-5} kgm^2

Ans. (A) *(a)* $\alpha = -\frac{\delta}{I}\theta$

Explanation: By Newton's third law, the torque τ_w, exerted by the wire on the external system is equal and opposite to the torque τ exerted system on the wire.

$\tau_w = -\delta\theta$.

As, $\tau = -\delta\theta$

where, δ is the torsion constant and $\tau = I\alpha$

where, I is the moment of inertia of the rod about an axis along the wire

so $I\alpha = -\delta\theta$

or $\alpha = -\frac{\delta}{I}\theta$

(B) *(b) 65.3 cm/sec*

Explanation: In SHM,

Displacement is given by,

$$x = A \sin(\omega t + \phi)$$

$$\frac{dx}{dt} = A\omega\sqrt{[1-\sin^2(\omega t + f)]}$$

$$= \omega\sqrt{(a^2 - x^2)}$$

$$= 2\pi\sqrt{(a^2 - x^2)}$$

$$= 65.3 \text{ cm/sec}$$

(C) *(a) 0.140 s and 3.14 rad*

Explanation: As

$$T = 2\pi\left(\frac{I}{\delta}\right)^{\frac{1}{2}}$$

$$= 2\pi\left(\frac{0.500 \text{ kg.m}^2}{1000 \text{ Nm/rad}}\right)^{\frac{1}{2}}$$

$$= 0.140 \text{ s}$$

Since, the bar is released from, $\theta = \pi$ at rest this must be the maximum angle and $\theta_A = 3.14$ rad.

(D) *(c) 1.154 s*

Explanation: The time period of oscillation of the torsional pendulum is given by,

$$T = 2\pi\sqrt{\frac{I}{K}}$$

where I is the moment of inertia and K is the torsional constant

$$T = 2\pi\sqrt{\frac{5}{15\pi^2}} = 1.154 \text{ s}$$

(E) (a) 8.11×10^{-5} kg m^2

Explanation: The graphs suggest that

$$T = 0.40 \text{ s and } K = \frac{6}{0.3} = 0.02 \text{ N.m/rad.}$$

With these values. Term $2\pi\sqrt{\frac{I}{K}}$ can be used to determine the rotational inertia:

$$I = \frac{KT^2}{4\pi^2} = \frac{0.02 \times (0.40)^2}{4 \times (3.14)^2}$$

$$= 8.11 \times 10^{-5} \text{ kg m}^2$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

15. **To keep accurate time, my uncle relies on the period of a pendulum of his clock. Assume that the clock is properly calibrated and then a mischievous child slides the pendulum's bob downward on the oscillating rod and the accuracy of the clock is disturbed. Analyze whether the clock is running on time or before time.**

Ans. With a longer length, the period of the pendulum increases. Thus, it takes longer to execute each swing, so each second according to the clock takes longer than an actual second. Thus, the clock runs slowly.

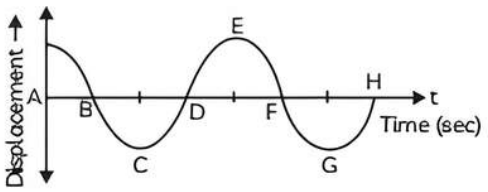
16. **How is the time period effected, if the amplitude of a simple pendulum is increased? [Delhi Gov. QB 2022]**

Ans. The distance to travel increases as the amplitude rises, but when the restoring force rises as well, the acceleration rises correspondingly. This implies that the mass may move faster and cover a bigger distance. Since these properties cancel each other, the amplitude has no impact on the period.

17. **Displacement versus time curve for a particle executing simple harmonic motion is shown in the figure. Identify the points marked at which:**

(A) Velocity of the oscillator is zero.

(B) Speed of the oscillator is maximum.



Ans. (A) Velocity of the oscillator is maximum, where the displacement is maximum i.e., at A, C, E and G.

(B) Speed of the oscillator will be maximum when no restoring force acts i.e., where the displacement of the oscillator is zero. i.e., at B, D, F and H.

18. **For a particle in SHM, the displacement x of the particle as a function of time t is given as x = A sin πt. Here x is in cm and t is in seconds. Let the time taken by the particle to travel from $x = 0$ to $x = \frac{A}{2}$ be t_1 and the time taken to travel from $x = \frac{A}{2}$ to $x = A$ be t_2. Find $\frac{t_1}{t_2}$. [Diksha]**

Ans. Given that: here, $x = 0$

at $t = 0$.

Also $\omega = \frac{2\pi}{T} = 2\pi$

$\therefore$ $T = 1$ s

At $t = t_1$

$x = \frac{A}{2}$.

Therefore, $\frac{A}{2} = A \sin(2\pi t_1)$

or $\frac{1}{2} = \sin(2\pi t_1)$

$\therefore$ $2\pi t_1 = \frac{\pi}{6}$

or $t_1 = \frac{1}{12}$ s

Time taken, from $x = 0$ to $x = A$ is $\frac{T}{4} = \frac{1}{4}$ s

or $t_1 + t_2 = \frac{T}{4}$

$= \frac{1}{4}$ s

or $t_2 = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$ s

Hence, $\frac{t_1}{t_2} = \frac{1/12}{1/6} = \frac{1}{2}$

19. **A linear platform that moves S.H.M. up and down around a mean position. Its oscillation period is 2π seconds. It is supported by a mass m. What will be the maximum amplitude that the platform can have, so that the mass resting on it does not fall off?**

Ans. When the platform reaches the highest point, the acceleration is maximum, which acts towards the mean position.

At the highest point,

$$R - mg = -ma$$

i.e., $$R = mg - ma.$$

The maximum acceleration of the platform,

$$a = \omega^2 A \text{ (Amplitude)}$$

$$R = mg - m\omega^2 A$$

For the object to rest on the platform, g should be less than A. The maximum value of A will give R = 0.

$$0 = mg - m\omega^2 A$$

$$A = \frac{g}{\omega^2}$$

$$A = \frac{9.8 \times T^2}{(4\pi^2)}$$

$$A = 9.8 \text{ meter}$$

20. **A simple harmonic motion of acceleration 'a' and displacement 'x' is represented by $a + 4\pi^2 x = 0$. What is the time period of S.H.M.?** **[Delhi Gov. QB 2022]**

Ans. The time period of S.H.M

$$a = -4\pi^2 x$$

$$= -\omega^2 x$$

$$\Rightarrow \quad \omega = 2\pi$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1 \text{ s}$$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

21. **Let's suppose a pale spring is connected with the mass of a known quantity and a meter scale for measuring time, then how one can determine the time period with the help of the given component in the absence of a clock.**

Ans. By suspending the known mass from one end of the spring, that another end is connected to a rigid ceiling. Note the extension l in the spring with the help of a metre scale. If k is the spring constant of the spring,

then for the equilibrium position,

$$kl = mg$$

or $$\frac{m}{k} = \frac{l}{g}$$

Time period of the loaded spring,

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{...(i)}$$

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \text{...(ii)}$$

Since, m and k are known, T can be calculated.

22. **Show that, the motion of a particle represented by, $y = \sin \omega t - \cos \omega t$ is simply harmonic with the period of $\frac{2\pi}{\omega}$.**

[NCERT Exemplar]

Ans. A function will represent S.H.M. if it can be written uniquely in the form of $A \cos\left(\frac{2\pi}{T}t + \phi\right)$ or $A \sin\left(\frac{2\pi}{T}t + \phi\right)$

Now, $$y = \sin \omega t - \cos \omega t$$

$$y = \sqrt{2}\left[\sin \omega t \frac{1}{\sqrt{2}} - \cos \omega t \frac{1}{\sqrt{2}}\right]$$

$$y = \sqrt{2}\left[\sin \omega t \cos\frac{\pi}{4} - \cos \omega t \sin\frac{\pi}{4}\right]$$

$$y = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

Comparing with standard SHM,

$$y = A \sin\left(\frac{2\pi}{T}t + \phi\right)$$

We get, $$\omega = \frac{2\pi}{T}$$

or $$T = \frac{2\pi}{\omega}$$

23. **The length of a simple pendulum is increased by 44%. What is the percentage change in the time period of the pendulum?** **[Diksha]**

Ans. Times period of a simple pendulum is directly proportional to the square root of the length of the pendulum.

The relation for time period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$ ...(i)

$T \propto \sqrt{l}$ ...(ii)

or $T \propto \sqrt{l'}$ ...(iii)

(Given, $l' = 1 + 44\%$ of $1 = 1 + \frac{44}{100}1 = \frac{144}{100}$)

Putting the given value of l' in eqn. (iii).

We get $T' = 2\pi\sqrt{\frac{144}{100}}$ ...(iv)

From eqn. (ii) and (iv), we get

$$\frac{T'}{T} = \sqrt{\frac{144}{100}} = \frac{12}{10}$$

Hence, $T' = \frac{12}{10}T$

Now percentage change in time period is

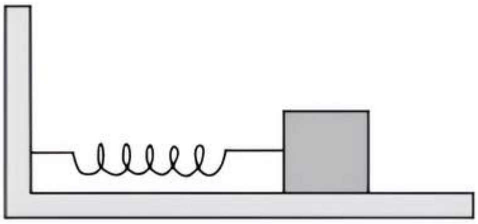
$$\left(\frac{T'-T}{T}\times 100\right) = \left(\left(\frac{\frac{12}{10}T - T}{T}\right)\times 100\right)$$

$$\left(\frac{T'-T}{T}\right)\times 100 = \frac{2}{10}\times 100 = 20\%$$

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

24. A spring having a spring constant 1500 Nm^{-1} is mounted on a horizontal table as shown. A mass of 5.0 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 0.2 cm and released. Determine:



(A) The frequency of oscillations.

(B) The maximum acceleration of the mass,

(C) the maximum speed of the mass.

Ans. Given that,

Spring constant,

$$k = 1500\ Nm^{-1}$$

mass, $m = 5$ kg

The distance at which mass is pulled from its equilibrium, $x = 0.2$ cm

(A) Frequency,

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$= \frac{1}{2\times 3.14}\sqrt{\frac{1500}{5}}$$

$$f = 2.75\ Hz$$

(B) The acceleration is given by,

$$a = -\omega^2 x$$

$$= -\frac{k}{m}x$$

or $|a_{max}| = \frac{k}{m}|x_{max}|$

i.e., acceleration a will be maximum, when x is maximum.

i.e., $x_{max} = A = 0.02$ m

$\therefore$ $a = \frac{1500}{5}\times 0.02 = 6.0\ ms^{-1}$

(C) The maximum speed of the mass is given by,

$$v = A\omega$$

$$= A\sqrt{\frac{k}{m}}$$

$$= 0.02\times\sqrt{\frac{1500}{5}}$$

$$= 0.34\ ms^{-1}$$

25. A mass of 2 kg is attached to the spring of spring constant 50 N-m^{-1}. The block is pulled to a distance of 5 cm from its equilibrium position at $x = 0$ on a horizontal frictionless surface from rest at $t = 0$. Write the expression for its displacement at anytime t.

[NCERT Exemplar]

Ans. Let the mass m attached to the spring oscillate in SHM with an amplitude 5 cm according to the question.

$$m = 2\ kg\ k = 50\ Nm^{-1}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{2}}$$

$$= 5\ radian/sec$$

Assume the displacement function is,

$$y(t) = A\sin(\omega t + \phi).$$

ϕ = initial phase

At t = 0,

$$y(t) = +A$$
$$y(t) = A\sin(\omega \times 0 + \phi)$$
$$= +A \text{ or } \sin(0+\phi) = 1$$
$$\sin\phi = 1$$

or $$\sin\phi = \sin\frac{\pi}{2}$$

$$\phi = \frac{\pi}{2}$$

So the desired equation becomes,

$$y(t) = A\sin(\omega t + \frac{\pi}{2})$$

Put A = 5,

$$\omega = 5 \text{ radian per second}$$

26. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released, so that the mass executes simple harmonic oscillations with a time period T. If the mass is increased by *m* then the time period becomes (5T/4). What will be the ratio of *m* and M? [Diksha]

Ans. As we know,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\frac{5T}{4} = 2\pi\sqrt{\frac{M+m}{k}}$$

$$\frac{5}{4}\left(2\pi\sqrt{\frac{m}{k}}\right) = 2\pi\sqrt{\frac{M+m}{k}}$$

On squaring both sides,

$$\frac{25M}{16k} = \frac{M+m}{k}$$

$$\frac{m}{M} = \frac{9}{16}$$

27. The length of a simple pendulum executing SHM is increased by 21%. By what % time period of the pendulum increases? [Delhi Gov. QB 2022]

Ans. Given, $\frac{l_2}{l_1} = 1.21$

The time period of the simple pendulum executing simple harmonic motion is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}} = \sqrt{1.21}$$

$$T_2 = 1.1T_1$$

The % increase in time period

$$= \frac{T_2 - T_1}{T_1} \times 100$$

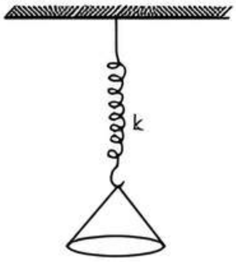
% increase

$$\frac{1.1T_1 - T_1}{T_1} \times 100 = 10\%$$

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

28. A robust disc of mass *m* is dropped from a height H onto a pan suspended by a spring with a spring constant *k*. Determine the amplitude of oscillation if the disc does not rebound and the pan is massless.



Ans. Velocity of the disc just before the collision,

$$u = \sqrt{2gH}$$

As the pan is massless, the velocity of the "pan + mass" system remains the same.

Now the new system has an equilibrium position *mg/k* below the natural length.

Energy of SHM as,

$$\frac{1}{2}mu^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{1}{2}kA^2$$

$$\frac{1}{2}m.2gH + \frac{m^2g^2}{2k} = \frac{1}{2}kA^2$$

$$A^2 = \frac{2mgH}{k} + \frac{m^2g^2}{k^2}$$

$$A^2 = \frac{m^2g^2}{k^2}\left(1 + \frac{2HK}{mg}\right)$$

$$A = \frac{mg}{k}\left(1 + \frac{2HK}{mg}\right)^{\frac{1}{2}}$$

29. **A tunnel is dug through the center of the earth. Show that a body of mass m when dropped from rest, from one end of the tunnel will execute the simple harmonic motion.** **[NCERT Exemplar]**

Ans. As the acceleration due to the gravity of the earth inside the earth is g'

$$g' = g\left(1-\frac{d}{R}\right)$$

$$= g\left[\frac{R-d}{R}\right]$$

$$R-d = y'$$

$$g' = g\frac{y}{R}$$

Force on the body at depth d is.

$$F = -mg'$$

$$= -mg\frac{y}{R}$$

$$F \propto (-y)$$

So, the motion of the body in the tunnel is SHM. For a period, we can write.

$$ma = -mg'$$

$$a = -\frac{g}{R}y$$

$$-\omega^2 y = -\frac{g}{R}y$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{R}}$$

or $$T = 2\pi\sqrt{\frac{R}{g}}$$

30. **A cylindrical log of wood of height h and area of cross-section A floats in water. It is pressed and then released, showing that the log would execute SHM with a time period $T = 2\pi\sqrt{\frac{m}{A\rho g}}$. Where, m is the mass of the body and ρ is density of the liquid.** **[NCERT Exemplar]**

Ans. When a log is pressed downward into the liquid than an upward buoyant force (B.F.) action moves the block upward and due to inertia it moves upward from its mean position due to inertia and then again comes down due to gravity.

So, net restoring force on the block

= Buoyant force − mg

V = volume of liquid displaced by block

Let when block floats then,

$$mg = \text{Buoyant force or } mg$$

$$= V\rho g$$

$$mg = Ax_0\rho g \qquad \text{...(i)}$$

A = area of cross-section

x_0 = height of block liquid

Let, x height again dip in liquid when pressed into water total height of the block in water

$$h = (x + x_0)$$

So, net restoring force

$$= [A(x + x_0)]\rho g - mg$$

$$F_{restoring} = A x_0 \rho g + Ax\rho g - Ax_0 \rho g$$

from equation (i)

$$F_{restoring} = -Ax\rho g$$

(as Buoyant force is upward and x is downward)

$$F_{restoring} \propto -x$$

So motion is SHM.

here $k = A\rho g$

$$a = -\omega^2 x.$$

$$\omega^2 = \frac{k}{m}.$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$F_{restoring} = -A\rho g x$$

$$ma = -A\rho g x$$

$$a = -\frac{A\rho g x}{m}.$$

$$-\omega^2 x = \frac{-A\rho g x}{m}$$

$$\omega^2 = \frac{-A\rho g}{m}$$

$$k = A\rho g$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{A\rho g}{m}$$

$$\frac{T}{2\pi} = \sqrt{\frac{m}{A\rho g}}$$

$$T = 2\pi\sqrt{\frac{m}{A\rho g}}$$

NUMERICAL Type Questions

31. What is the maximum value of acceleration, when a body is vibrating with SHM of amplitude 25 cm and frequency 6 Hz? (2m)

Ans. As we know, maximum acceleration.

$$a = \omega^2 A$$
$$= (2\pi v)^2 A$$
$$= 4\pi^2 v^2 A$$
$$= 4 \times (3.14)^2 \times (6)^2 \times 25 \times 10^{-2}$$
$$= 354.94 \text{ ms}^{-2}$$

32. What fraction of the total energy is:

(A) Kinetic energy

(B) Potential energy when the displacement in SHM is one-half the amplitude x_m?

(C) At what amplitude of displacement, is the system's energy half of the kinetic energy and half of potential energy?(3m)

Ans. (A) Fraction of total energy which is kinetic.

$$\frac{K.E}{T.E} = \frac{\frac{1}{2}m\omega^2(a^2-b^2)}{\frac{1}{2}m\omega^2a^2}$$

$$\frac{a^2-y^2}{a^2} = \frac{a^2-\frac{a^2}{4}}{a^2}$$

$$= \frac{\frac{3a^2}{4}}{a^2} = \frac{3}{4}\text{th}$$

(B) Fraction of total energy which is potential

$$\frac{P.E}{T.E} = \frac{\frac{1}{2}m\omega^2y^2}{\frac{1}{2}m\omega^2a^2} = \frac{y^2}{a^2}$$

$$\frac{\frac{a^2}{4}}{a^2} = \frac{1}{4}\text{th} \qquad \left[y = \frac{a}{2}\right]$$

(C) When the total energy is half of the kinetic and half potential with K.E = P.E.

$$\frac{1}{2}m\omega^2(a^2-y^2) = \frac{1}{2}m\omega^2y^2$$
$$a^2-y^2 = y^2$$
$$y = \frac{a}{\sqrt{2}}$$

33. The acceleration due to gravity on the surface of the moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the moon, if its time period on the earth is 3.5 s? [g = 9.8 ms^{-2}] (3m)

Ans. Acceleration due to gravity on the surface of the moon, $g = 1.7 \text{ ms}^{-2}$

Acceleration due to gravity on the surface of the earth, $g = 9.8 \text{ ms}^{-2}$

Time period of a simple pendulum on earth,

$$T = 3.5 \text{ s}$$

$$T = 2\pi\sqrt{\frac{1}{g}}$$

where,

1 is the length of the pendulum

$$\therefore \quad 1 = \frac{T^2}{(2\pi)^2} \times 2$$

$$= \frac{(3.5)^2}{4\times(3.14)^2} \times 9.8 \text{ m}$$

The length of the pendulum remains constant

On the moon's surface, time period,

$$T' = 2\pi\sqrt{\frac{1}{g}}$$

$$= 2\pi\sqrt{\frac{\frac{(3.5)^2}{4\times(3.14)^2}\times 9.8}{1.7}}$$

$$= 8.4 \text{ s}$$

Hence, the time period of the simple pendulum on the surface of the moon is 8.4 s.

DIFFERENT TYPES OF OSCILLATIONS 2

TOPIC 1

ENERGY IN SIMPLE HARMONIC MOTION

When a system, at rest, is shifted out of its equilibrium position by doing work on it, it gains potential energy. when it is released, it begins to move with velocity and gains kinetic energy.

If m is the mass of the system executing SHM, then the kinetic energy of the system at any given time is

$$K.E. = \frac{1}{2} mv^2$$

Putting the value of velocity in the above equation,

$$K.E. = \frac{1}{2} m - \omega A \sin(\omega t + \phi)^2$$

$$K.E. = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \phi)$$

According to the equation,

the Kinetic Energy of the system varies periodically.

i.e., it is maximum ($= \frac{1}{2} m\omega^2 A^2$) at the maximum value of velocity ($\pm \omega A$), and displacement is zero at this time.

When displacement is maximum ($\pm$ A), velocity of SHM is zero, and thus, kinetic energy is also zero and all energy is potential at these extreme points, where kinetic energy, K.E. = 0.

The energy is partly kinetic and partly potential at intermediate positions between 0 and A.

To calculate potential energy at any point in time, consider x to be the system's displacement from equilibrium at any time t.

We know that a system's potential energy is given by the amount of work required to move it from position 0 to position x under the action of applied force.

In this case, the force applied to the system must be just enough to counteract the restoring force $-kx$, *i.e.*, it must be equal to kx.

Now work required to give infinitesimal displacement is, $dx = kx\,dx$.

Thus, total work required to displace the system from 0 to x is

$$= \int_0^x kx\,dx$$

$$= \frac{1}{2} kx^2$$

So, $$P.E. = -kx^2$$

$$= \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \phi)$$

Thus, total energy

$$= K.E. + P.E.$$

$$= \frac{1}{2} m\omega^2 A^2 [\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

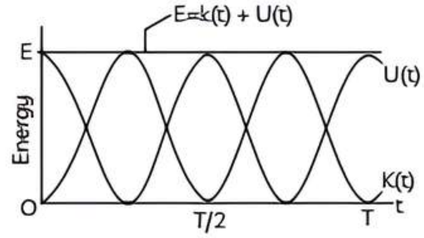
Or $$E = \frac{1}{2} m\omega^2 A^2$$

As displacement is regained after every half cycle, the oscillator's total energy remains constant. If no energy is dissipated, all potential energy is converted to kinetic energy and vice versa.

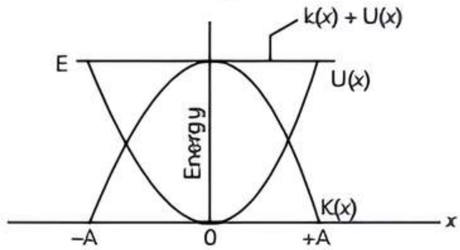
Important

➥ *Frequency of total energy is zero because it remains constant.*

The graph given below depicts the variation of kinetic and potential energy of a harmonic oscillator with time, with phase set to zero for simplicity.



(a)



(b)

Kinetic energy, potential energy and total energy as a function of time [shown in (a)] and displacement [shown in (b)] of a particle in SHM. The kinetic energy and potential energy both repeat after a period $\frac{T}{2}$.

The total energy remains constant at all t or x.

⚠ Caution

➥ *Students should know that the frequency of oscillation of potential energy and kinetic energy is twice as that of displacement or velocity or acceleration of a particle executing S.H.M.*

Example 2.1: **Show that for a particle in linear SHM. the average kinetic energy over a period of oscillation equals the average potential energy over the same period.**

Ans. Let the particle executing SHM start oscillating from its mean position. Then displacement equation is:

$$x = A \sin \omega t$$

Particle velocity,

$$v = A\omega \cos \omega t$$

Instantaneous K.E.

$$K = \frac{1}{2} mv^2$$

$$= \frac{1}{2} mA^2 \omega^2 \cos^2 \omega t$$

Average value of K.E. over one complete cycle

$$K_{av} = \frac{1}{T} \int_0^2 \frac{1}{2} mA^2 \omega^2 \cos^2 \omega t dt$$

$$= \frac{mA^2\omega^2}{2T} \int_0^2 \cos^2 \omega t dt$$

$$= \frac{mA^2\omega^2}{2T} \int_0^2 \cos^2 \omega t dt$$

$$= \frac{mA^2\omega^2}{2T} \int_0^2 \frac{1+\cos^2 \omega T}{2} dt$$

$$= \frac{mA^2\omega^2}{4T} \left[t + \frac{\sin^2 \omega t}{2\omega} \right]_0^T$$

$$= \frac{mA^2\omega^2}{4T} \left[(T-0) + \frac{(\sin^2 \omega t - \sin 0)}{2\omega} \right]$$

$$= \frac{1}{4} mA^2\omega^2$$

Again instantaneous P.E.

$$U = \frac{1}{2} kx^2$$

$$= \frac{1}{2} k\omega^2 x^2$$

$$= \frac{1}{2} k\omega^2 A^2 \sin^2 \omega t$$

Average value of P.E. over one complete cycle.

$$U = \frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2$$

$$= \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$$

Average value of P.E. over one complete cycle.

$$U_{av} = \frac{1}{T} \int_0^T \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$$

$$= \frac{m\omega^2 A^2}{2T} \int_0^T \sin^2 \omega t dt$$

$$= \frac{m\omega^2 A^2}{2T} \int_0^T \frac{(1-\cos^2 \omega t)}{2} dt$$

$$= \frac{m\omega^2 A^2}{4T} \left[t - \frac{\sin^2 \omega t}{2\omega} \right]_0^T$$

$$= \frac{m\omega^2 A^2}{4T} \left[(T-0) - \frac{(\sin 2\omega T - \sin 0)}{2\omega} \right]_0^T$$

$$= \frac{1}{4} m\omega^2 A^2$$

Simple comparison of both equations.

$$K_{av} = U_{av} = \frac{1}{4} m\omega^2 A^2$$

Example 2.2: **A body of mass 3 kg is executing SHM given by, $x = 15 \cos (100t + \frac{\pi}{4})$ cm. What is the**

(A) velocity,

(B) acceleration and

(C) maximum kinetic energy?

Ans. **(A)** Given.

$$x = 15 \cos (100t + \frac{\pi}{4}) \text{ cm.}$$

$$\text{Velocity, } v = \frac{dx}{dt}$$

$$= -1500 \sin (100t + \frac{\pi}{4})$$

(B) Acceleration.

$$a = \frac{d^2x}{dt^2}$$

$$= -150000 \cos (100t + \frac{\pi}{4})$$

(C) K.E. is maximum when P. E. is minimum (= 0) and the total energy is constant.

Maximum K.E.

$$K = \frac{1}{2} m\omega^2 a^2$$

$$K = \frac{1}{2} \times 3 \times (100)^2 \times (0.15)^2$$

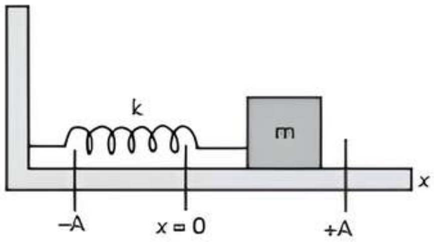
$$K = 337.5 \text{ J}$$

TOPIC 2

SOME SYSTEMS EXECUTING SIMPLE HARMONIC MOTION

Oscillations due to a Spring

Let the time period of a spring-mass system oscillating on a smooth horizontal surface, as illustrated in the figure.



According to the figure, the linear simple harmonic oscillator consisting of a block of mass m attached to a spring. The block moves over a frictionless surface. The box, when pulled or pushed and released, executes simple harmonic motion.

The spring is relaxed in the equilibrium position. When the block is moved x distance to the right, it experiences a net restoring force, $F = -kx$ towards the left.

The negative sign indicates that the restoring force is always diametrically opposed to the smooth displacement. In other words, if x is positive and F is negative, the force is directed to the left.

When x is negative and F is positive, the force tends to return the block to its equilibrium position of, $x = 0$.

$$F = -kx$$

Applying Newton's second law,

$$F = m\frac{d^2x}{dt^2}$$

$$= -kx \text{ or } \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x$$

$$= 0$$

Comparing the above equation with,

$$a = \frac{d^2x}{dt^2}$$

$$= -\omega^2 x$$

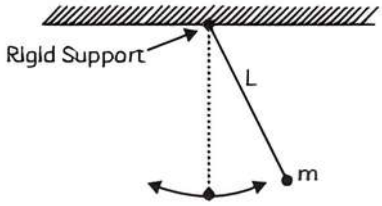
we get, $$\omega^2 = \frac{k}{m}$$

or $$T = 2\pi\sqrt{\frac{m}{k}}$$

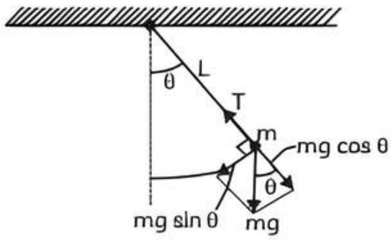
The amplitude has no effect on the time period. For a given spring constant, the period of the block increases with mass, implying that a more massive block oscillates slowly. As k increases, the period for a given block decreases. A stiffer spring causes a faster oscillations.

The Simple Pendulum

A simple pendulum is a bob of mass 'm' suspended at the free end of an unstretchable massless string of length 'l', while the other end is fixed.



(a)



(b)

Figure (a) shows, a bob oscillating about its mean position. Figure (b) shows the radial force $T = mg\cos\theta$ provides centripetal force but no torque about the support. The tangential force $mg\sin\theta$ provides the restoring torque.

The pendulum oscillates around the equilibrium/mean position O due to the influence of the tangential component of weight, which acts in the opposite direction of the bob's displacement and attempts to return it to the mean position. As a result, for small angles of displacement (<OPA), the motion of a simple pendulum can be modelled as SHM with time period is given by,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

$$= 2\pi\sqrt{\frac{l}{g}}$$

Where, $I = ml^2$

= moment of inertia

A second's pendulum is a simple pendulum with a time period of 2 seconds.

Important

➥ *The force constant k of a stiffer spring is higher than that of a soft spring. So, the time period of a stiffer spring is less than that of a soft spring.*

Example 2.3: You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of

(A) the spring constant k and

(B) the damping constant b for the spring and shock absorber system of one wheel. Assuming that, each wheel supports 750 kg.

[g = 10 m/s^2].

Ans. (A) Here, Mass,

$$M = 300 \text{ kg.}$$

Displacement,

$$x = 15 \text{ cm} = 0.15 \text{ m.}$$

$$g = 10 \text{ m/s}^2.$$

There are four spring systems. If k is the spring constant of each spring, then total spring constant of all four springs in parallel is:

$$k_P = 4k$$

$$M_g = k_p x = 4kx$$

$$k = \frac{M_g}{4x}$$

$$= \frac{3000 \times 10}{4 \times 0.15} = 5 \times 10^4 \text{ N}$$

(B) For each spring system supporting 750 kg of weight,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2 \times 3.14 \times \sqrt{\frac{750}{5 \times 10^4}}$$

$$T = 0.77 \text{ sec}$$

Using, $x = x_0\, e^{\frac{-bt}{2m}}$

We get,

$$\frac{50}{100}x_0 = x_0\, e^{\frac{-b \times 0.77}{2 \times 750}}$$

or $e^{\frac{0.77b}{1500}} = 2$

Taking the logarithm of both sides,

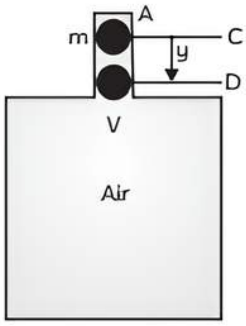
$$\frac{0.77b}{1500} = \log_e 2$$

$$= 2.303 \log 2$$

$$b = \frac{1500}{0.77} \times 2.303 \times 0.3010$$

$$b = 1350.4 \text{ kg s}^{-1}$$

Example 2.4: An air chamber of volume V has a neck area of cross-section into which a ball of mass m just fits and can move up and down without any friction (fig.). Show that, when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations, assuming pressure-volume variations of air to be isothermal.



[NCERT]

Ans. Given, an air chamber of volume V with a long neck of uniform area of cross-section A, and a frictionless ball of mass m fitted smoothly in the neck at position C (from the figure). The pressure of air below the ball inside the chamber is equal to the atmospheric pressure. Increasing the pressure on the ball by a little amount P, So that the ball is depressed to position D, where CD = y. Then there will be a decrease in volume and hence increase in pressure of air inside the chamber. The decrease in volume of the air inside the chamber, $\Delta y = Ay$

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}}$$

$$= \frac{\Delta V}{V} = \frac{Ay}{V}$$

Bulk Modulus of elasticity, E, will be,

$$E = \frac{\text{stress (or increase in pressure)}}{\text{volumetric strain}}$$

$$= \frac{-P}{\frac{Ay}{V}} = \frac{-PV}{Ay}$$

Here, a negative sign shows that the increase in pressure will decrease the volume of air in the chamber.

Now, $P = \frac{-EAy}{V}$

Due to this excess pressure, the restoring force acting on the ball is,

$$F = P \times A$$

$$= \frac{-EAy}{V}.$$

$$F = \frac{-EA^2}{V} y \qquad \text{...(i)}$$

Since $F \propto -y$ and negative sign shows that the force is directed towards equilibrium position. If the applied increased pressure is removed from the ball, the ball will start executing linear SHM in the neck of the chamber with C as the mean position.

In SHM, the restoring force,

$$F = -ky \qquad \text{...(ii)}$$

Comparing eqn. (i) and (ii) and we have,

Spring factor, $k = \frac{EA^2}{V}$

Here, inertial factor = mass of ball = m.

Time Period,

$$T = 2\pi\sqrt{\frac{\text{inertial factor}}{\text{spring factor}}}$$

$$= 2\pi\sqrt{\frac{m}{\frac{EA^2}{V}}}$$

$$= \frac{2\pi}{A}\sqrt{\frac{mV}{E}}$$

Frequency, $\nu = \frac{1}{T} = \frac{A}{2\pi}\sqrt{\frac{E}{mV}}$

If the ball oscillates in the neck of the chamber under isothermal conditions, then E = P = pressure of air inside the chamber, when the ball is at the equilibrium position. If the ball oscillates in the neck of the chamber under adiabatic conditions,

then, $E = gP$

where $g = \frac{C_p}{C_v}$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. **SHM is performed by a particle on a 24 cm-long line. Its K.E. and P.E. will be equal when it's at the distance from the mean position:**

(a) 3.145 cm **(b) 6.134 cm**

(c) 7456 cm **(d) 8.485 cm**

Ans. *(d) 8.485 cm*

Explanation: Let A be the amplitude of the given SHM.

Length of the line, in which SHM executes, will be twice of the amplitude of SHM.

So, $2A = 24$

$\Rightarrow A = 12$ cm

K.E. = P.E.

$$\frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} m\omega^2$$

$$A^2 - x^2 = x^2$$

$$2x^2 = A^2$$

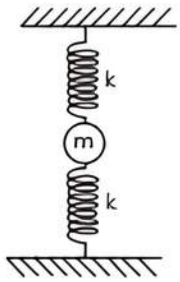
$$x = \frac{A}{\sqrt{2}} = \frac{12}{\sqrt{2}}$$

$$= 8.485 \text{ cm}$$

Related Theory

➥ *The frequency of oscillation of potential energy and kinetic energy is twice as that of displacement or velocity or acceleration of a particle executing S.H.M.*

2. **When a mass M is suspended from a spring with a force constant k, the frequency of vibration f is obtained. If a second spring is arranged as shown in the figure, the frequency will be:**



(a) $\frac{f}{\sqrt{2}}$ **(b)** $2f$

(c) f **(d)** $\frac{f}{\sqrt{2}}$

Ans. *(d)* $\frac{f}{\sqrt{2}}$

Explanation: Let the time period of SHM is 1 when spring and mass are connected.

Spring constant, k

Attached mass, m

So, frequency is,

$$f = 2\pi\sqrt{\frac{k}{m}}$$

As shown in the figure, if another spring is added in parallel. Both springs are identical new spring constant for two parallel springs.

$$k_{new} = k + k = 2k$$

Time period of oscillation,

$$f_{new} = 2\pi\sqrt{\frac{m}{k_{new}}}$$

$$= 2\pi\sqrt{\frac{m}{2k}}$$

$$f_{new} = \frac{1}{\sqrt{2}}$$

$$2\pi\sqrt{\frac{m}{k}} = \frac{f}{\sqrt{2}}$$

Hence, new frequency is $\frac{f}{\sqrt{2}}$.

Related Theory

➥ *When a tuning fork is struck against a rubber pad, the prongs begin to execute free vibration. When the stem of vibrating tuning fork is pressed against the top of table, then the table will suffer forced vibration.*

3. A mass m body is attached to the lower end of a spring, the upper end of which is fixed. The mass of the spring is insignificant. When mass m is slightly dragged down and released, it oscillates with a time period of 7 s. When the mass m is increased by one kilogram, the time period of the oscillations is increased to 6 s. Then the value of m in kilogram is:

(a) 2.76 kg **(b) 2 kg**

(c) 1 kg **(d) 3.077 kg**

Ans. *(a) 2.76 kg*

Explanation: For SHM of a hanging mass by a spring,

$$F = -k_s x$$

where,

$$k_s = m\omega^2$$

$$\omega = \sqrt{\frac{k_s}{m}}$$

Hence time period of SHM

$$T = \frac{2\pi}{\omega}$$

$$= 2\pi\sqrt{\frac{m}{k_s}}$$

$$T \propto \sqrt{m}$$

Hence,

$$\frac{7}{6} = \sqrt{\frac{m+1}{m}}$$

$$m = 2.76 \text{ kg}$$

⚠ Caution

➥ *Students should know that if a spring of a spring constant k is divided into n equal parts, the spring constant of each part becomes nk and time period becomes $\frac{1}{\sqrt{n}}$ times.*

4. Two simple harmonic motions are represented by the equations, $y_1 = 3.1 \sin(10\pi t + \frac{\pi}{3})$ and $y_2 = 3.1 \cos 10\pi t$. The phase difference of particle 1's velocity with respect to particle 2's velocity at $t = 0$ is:

(a) $\frac{\pi}{6}$ **(b) $\frac{\pi}{4}$**

(c) $-\frac{\pi}{6}$ **(d) $-\frac{\pi}{4}$**

Ans. *(c) $-\frac{\pi}{6}$*

Explanation: Given that:

$$y_1 = 3.1\sin(10\pi t + \frac{\pi}{3})$$

$$y_2 = 3.1 \cos 10\pi t$$

$$= 3.1 \sin\left(\frac{\pi}{2} + \pi t\right)$$

$$= 3.1 \sin\left(\frac{\pi}{2} + \pi t\right)$$

Now, to find the velocity of a particle, differentiate both equations with respect to time.

$$\frac{dy_1}{dt} = v_1$$

$$= 3.1 \times 10\pi \cos\left(\pi t + \frac{\pi}{2}\right)$$

Similarly for 2nd equation,

$$\frac{dy_2}{dt} = v_2$$

$$= 3.1 \times \pi \cos\left(\pi t + \frac{\pi}{2}\right)$$

$$= 3.1\pi \cos\left(\pi t + \frac{\pi}{2}\right)$$

If equation $x = A \sin(\omega t + \phi)$ is given then, at $t = 0$ phase of motion is ϕ,

similarly at $t = 0$ phase of 1st particle velocity is $\frac{\pi}{3}$

at $t = 0$ phase of velocity of 2nd particle is $\frac{\pi}{2}$.

Now the phase difference = phase of 1st particle at $t = 0$ – phase of 2nd particle at $t = 0$

$$\phi = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$$

5. **A signal generator is made up of a block connected to a spring (k = 500 N/m). At time t, the block's position (as measured from the system's equilibrium location), velocity and acceleration are x = 0.200 m, v = –15.4 m/s, and a = –132 m/s^2. The amplitude of the motion is:**

(a) 0.435 m **(b) 1 m**

(c) 0.052 m **(d) 0.625 m**

Ans. *(d) 0.625 m*

Explanation: From the equation.

$$a = -\omega^2 x.$$

$$\omega = \sqrt{-\frac{a}{x}}$$

$$= \sqrt{\frac{132 \text{ m/s}^2}{0.200}}$$

$$= 25.6 \text{ Hz}$$

Therefore, $f = \dfrac{\omega}{2\pi}$

$$f = 4.074 \text{ Hz}$$

Equation, $\omega = \sqrt{\dfrac{k}{m}}$ provides a relation between ω and the mass:

$$\omega = \sqrt{\frac{k}{m}}$$

$$m = \frac{500 \text{ N/m}}{(25.6)^2}$$

$$m = 0.76 \text{ kg}$$

By energy conservation law, $\frac{1}{2}kx_m^2$ (the energy of the system at a turning point) is equal to the sum of kinetic and potential energies at the time t.

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 + \frac{1}{2}mx^2$$

$$x_m = \sqrt{\left(\frac{0.76}{500}\right)(-15.4)^2 + (0.200)^2}$$

$$x_m = 0.625 \text{ m}$$

6. **A 45 kg man stands on a channel, performing SHM in the vertical plane. The displacement from the mean position varies as y = 0.5 sin ($2\pi ft$) the value of f for which the man will experience weightlessness at its maximum:**

(a) $\dfrac{\sqrt{g}}{2\pi}$ **(b)** $\dfrac{\sqrt{2g}}{2\pi}$

(c) $4\pi g$ **(d)** $\dfrac{2\pi}{\sqrt{2g}}$

Ans. *(b)* $\dfrac{\sqrt{2g}}{2\pi}$

Explanation: $|a| = \omega^2 A = g$

for weightlessness at highest point

$$\omega = \sqrt{\frac{g}{A}} = 2\pi f$$

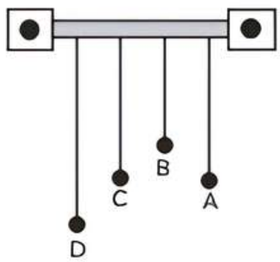
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{A}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{0.5}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{1/2}}$$

$$f = \frac{\sqrt{2g}}{2\pi}$$

7. **Four pendulums A, B, C and D are suspended from the same elastic support as shown in the figure. A and C are of the same length, while B is smaller than A and D is larger than A. If A is given a transverse displacement:**



(a) D will vibrate with maximum amplitude.

(b) C will vibrate with maximum amplitude.

(c) B will vibrate with maximum amplitude.

(d) All four will oscillate with equal amplitude. **[NCERT Exemplar]**

Ans. *(b) C will vibrate with maximum amplitude.*

Explanation: When pendulum vibrate with transverse vibration then,

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

l = length of pendulum A and C.

The disturbance produce in elastic rigid support of time period T, which is transmitted by support to all pendulum B, C and D, but the frequency or time period of C is the same as of A.

So a periodic force of period T produces resonance in C and C will vibrate with maximum as in resonance.

8. Which of the following functions of time represents simple harmonic motion?

(a) $e^{-\omega t}$ (b) $\sin \omega t - \cos \omega t$

(c) $\log \omega t$ (d) $\frac{1}{\log} \omega t$

Ans. *(b)* $\sin \omega t - \cos \omega t$

Explanation: From the given option (b).

$\sin \omega t - \cos \omega t$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin \omega t - \frac{1}{\sqrt{2}}\cos \omega t\right]$$

$$= \sqrt{2}\left[\sin \omega t \times \cos\frac{\pi}{4} - \cos \omega t \times \sin\frac{\pi}{4}\right]$$

$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

This function represents SHM as it can be written in the form: A sin ($\omega t + \phi$).

Its period is: $\frac{2\pi}{\omega}$.

9. Two equations of two SHM, y = A Sin ($\omega t - \alpha$) and y = B Cos ($\omega t - \alpha$). The phase difference between the two is:

(a) 0° (b) α°

(c) 90° (d) 180°

[Delhi Gov. QB 2022]

Ans. *(c) 90°*

Explanation:

$$x = A\sin(\omega t - \alpha) = A\cos(\omega t - \alpha - \frac{\pi}{2})$$

Another equation is

$$y = B\cos(\omega t - \alpha)$$

$\therefore$ The phase difference = $(x - y)$ at $t = 0$

$$A\cos(\omega.0 - \alpha - \frac{\pi}{2}) - B\cos(\omega.0 - \alpha)$$

$$= \frac{\pi}{2} = 90°$$

Assertion–Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

10. Assertion (A): For a harmonic oscillation, the plot of velocity and displacement is a straight line.

Reason (R): Velocity changes uniformly with displacement in simple harmonic motion

Ans. *(d) A is false and R is also false.*

Explanation: In a simple harmonic oscillator, the velocity is given by:

$$v = \omega\sqrt{a^2 - y^2}$$

$$v^2 = \omega^2 a^2 - \omega^2 y^2$$

Dividing both sides by $\omega^2 a^2$.

$$1 = \frac{v^2}{\omega^2 a^2} + \frac{y^2}{a^2}$$

This is an equation of the ellipse.

And the velocity does not change uniformly with displacement in harmonic motion.

11. Assertion (A): Time average K.E. and the time average P.E. are not exactly equal in simple pendulum.

Reason (R): Friction is not negligible in a simple pendulum.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: The sum of K.E. and P.E. is constant they are not equal. Since, their sum is constant at any instant, at some instant they may be equal. Near the mean position K.E. is maximum while P.E. is minimum. Near the extremes it is just the opposite. So, if the time interval is chosen near the mean position or near the extremes the average K.E. and P.E. will not be equal. The variations of the K.E. and P.E. are exactly similar in one time period except that K.E. is maximum at the mean position while P.E. is maximum at the extremes. So, the average P.E. and K.E. are equal in one time period.

12. Assertion (A): In SHM, kinetic energy is zero when potential energy is maximum.

Reason (R): In SHM, the kinetic and potential energies become equal when the displacement is $\frac{1}{\sqrt{2}}$ times the amplitude.

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: When the displacement of a particle executing SHM is y, then

$$KE = \frac{1}{2} m \omega^2 (a^2 - y^2)$$

and $$PE = \frac{1}{2} m \omega^2 y^2$$

For $$K.E = P.E$$

or $$2y^2 = a^2$$

Or $$y = \frac{a}{\sqrt{2}}$$

Since total energy remains constant throughout the motion, which is E = K.E + P.E. So, when P.E is maximum then K.E is zero and *vice versa.*

13. Assertion (A): The graph of total energy of a particle in SHM w.r.t. position is a straight line with zero slope.

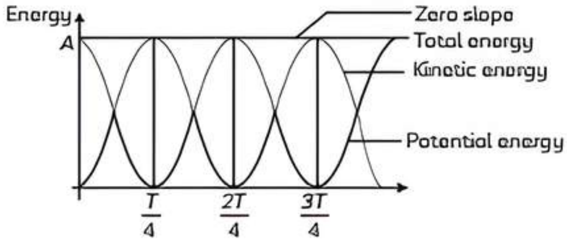
Reason (R): Total energy of the particle in SHM remains constant throughout its motion.

[Delhi Gov. QB 2022]

Ans. (*b*) *Both A and R are true and R is the correct explanation of A.*

Explanation: The total energy of S.H.M. = kinetic energy of particle + potential energy of particle.

The variation of total energy of the particle in SHM with time is shown in a graph.



CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

14. *If the bob of a vibrating simple pendulum is made of ice, then the period of swing of the simple pendulum will remain unchanged till the location of centre of gravity of the ice bob left after melting the ice remains at a fixed distance from the point of suspension. If the centre of a gravity of an ice bob after melting is raised upwards, then the effective length of a pendulum decreases and start time period of the swing decreases. If the centre of gravity shifts on the lower side, the time period of the swing increases.*

(A) What will happen to the time in a pendulum clock at hills or inside the mines?

(B) Length of a simple pendulum is infinite then why is time not infinite?

(C) If the mass of the pendulum is increased two-fold, then what will be the effect on the periodic time of the pendulum?

Ans. (A) As time is inversely proportional to the square root of gravity, time period will decrease with the value of *g*. The value of acceleration due to gravity is less at hills or in the mines than that of on the surface of Earth, the time period of simple pendulum increases at hills or inside the mines. Hence, the pendulum clock will be slowed down which means it will be losing time.

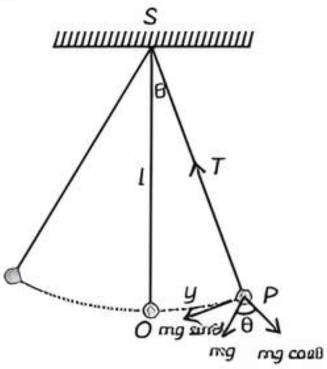
(B) The relation of time period for a simple pendulum is not valid if the effective length of a simple pendulum is more than the radius of the earth. This means that, T increases up to a certain limit only.

(C) There will be no change in the periodic time because the periodic time is independent of the mass but depends upon the length of the Pentagon and the acceleration due to gravity, which will not be affected if the mass of the pendulum is increased.

15. *An ideal simple pendulum consists of a heavy point mass body (bob) suspended by a weightless, inextensible and perfectly flexible string from a rigid support about which it is free to oscillate.*

But in reality, neither point mass nor weightless string exists, so we can never construct a simple pendulum strictly according to the definition.

Suppose a simple pendulum of length l is displaced through a small angle □ from it's mean (vertical) position. Consider m as of the bob is m and linear displacement from mean position is x.



[Delhi Gov. QB 2022]

(A) The period of a simple pendulum is doubled, when:

(a) its length is doubled
(b) the mass of the bob is doubled
(c) its length is made four times
(d) the mass of the bob and the length of the pendulum are doubled

(B) The period of oscillation of a simple pendulum of constant length at earth surface is T. Its period inside a mine is:

(a) greater than T
(b) less than T
(c) equal to T
(d) cannot be compared

(C) A pendulum suspended from the ceiling of a train has a period T, when the train is at rest. When the train is accelerating with a uniform acceleration *a*, the period of oscillation will:

(a) increase
(b) decrease
(c) remain unaffected
(d) become infinite

(D) Which of the following statements is not true? In the case of a simple pendulum for small amplitudes the period of oscillation is:

(a) directly proportional to square root of the length of the pendulum
(b) inversely proportional to the square root of the acceleration due to gravity
(c) dependent on the mass, size and material of the bob
(d) independent of the amplitude

(E) Assertion (A): The frequency of a second pendulum in an elevator moving up with an acceleration half the acceleration due to gravity is 0.612 Hz.

Reason (R): The frequency of a second pendulum does not depend upon acceleration due to gravity.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

Ans. (A) *(c) Its length is made four times*

Explanation: For a simple pendulum the time period of swing of a pendulum depends on the length of the string and acceleration due to gravity.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

When its length is made four times, then time period of simple pendulum is

$$T_1 = 2\pi\sqrt{\frac{4l}{g}} = 2\left(2\pi\sqrt{\frac{l}{g}}\right) = 2T$$

The period of a simple pendulum is doubled, when its length is made four times.

(B) *(a) greater than* T

Explanation: Value of *g* decreases on going below the earth's surface. The time period (T) of a simple pendulum of length, *l* and acceleration due to gravity *g* is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \text{...(i)}$$

$$\Rightarrow \qquad T = \frac{l}{\sqrt{g}}$$

Let mine be at a depth *h* below the surface of the earth having radius *R*, then

$$g' = g\left(1 - \frac{h}{g}\right)$$

Hence, *g* decreases.
Therefore, from eqn. (i), T increases.

(C) *(b) decrease*

Explanation: The pendulum is forced in the opposite direction of the train's motion while the train is going with acceleration. As a result, the pendulum's effective acceleration will rise. Due to the fact that the time period is inversely related to the square root of the effective acceleration, the time period will shorten.

(D) *(c) dependent on the mass, size and material of the bob*

Explanation: It is abundantly obvious from the above mentioned equation that a simple pendulum's period is exactly proportional to its square root length and inversely proportional to its square root acceleration owing to gravity. As a result, option (*a*) and (*b*) are right.

As long as a simple pendulum's motion is simple harmonic, its period is independent of its amplitude. However, it is too large, $\sin\theta \neq \theta$, then the motion will oscillate instead of being simple harmonic. Therefore, option (*d*) is right.

It is abundantly obvious from the afore-mentioned equation that the time period of a basic pendulum is independent of the mass of the bob. Option (c) is thus, erroneous.

(E) *(c) A is true but R is false.*

Explanation: Frequency of second pendulum $n = (1/2)s^{-1}$. When elevator is moving upwards with acceleration $\frac{g}{2}$, the effective acceleration due to gravity is:

$$g = g + a = g + \frac{g}{2} = \frac{3g}{2}$$

As $$n = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \text{ so } n^2 \propto g.$$

∴

$$\frac{n_1^2}{n_2^2} = \frac{g_1}{g} = \frac{3g/2}{g} = \frac{3}{2}$$

or $$\frac{n_1}{n} = \sqrt{\frac{3}{2}} = 1.225$$

or, $$n_1 = 1.225n$$

$$= 1.225 \times \left(\frac{1}{2}\right) = 0.612 \text{ s}^{-1}.$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

16. A simple pendulum is suspended from the elevator ceiling. What is the frequency of the pendulum's oscillation, if the elevator falls freely under gravity?

Ans. The frequency of oscillation of the pendulum,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

During free fall effective value of 'g' = 0 therefore, $\nu = 0$.

17. The graph of energy (K.E. or P.E.) versus time then shows that for one complete oscillation both K.E. and P.E. achieve two complete oscillations. Then how are their frequencies related?

Ans. P.E. or K.E. completes two vibrations in a time during which S.H.M. completes one vibration. So, we can say that the frequency of oscillation is double the frequency of change in K.E. as well as P.E. of the body in S.H.M.

18. There are two springs, one delicate and another hard or stout one. For which spring, the frequency of the oscillator will be more? [Delhi Gov. QB 2022]

Ans. We have,

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

So, when a mass m is applied to a firm spring, In comparison to the delicate one, the extension l will be smaller. As a result, the hard spring will oscillate more frequently and less frequently if time is given. Hence, the tougher spring's frequency of oscillation will be higher than that of the delicate spring because its force constant, k, is larger.

19. At what time from mean position of a body executive S.H.M. kinetic energy and potential energy will be equal? [Delhi Gov. QB 2022]

Ans. Maximum potential energy is zero while kinetic energy is present. Potential energy will become equal to kinetic energy when it is reduced to half of its greatest value.

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

20. A pendulum with a time period of 1 s is losing energy due to damping. At a certain time, its energy is 50 J. If after completing 20 oscillations, its energy has become 10 J, find its damping constant (in s^{-1}). [Diksha]

Ans. The amplitude of the damped oscillations at any time t is given by,

$$A = A_0 e^{-\frac{bt}{2m}}$$

So, energy at any time t is given by,

$$E = \frac{1}{2}KA_0^2 e^{-\frac{bt}{m}}$$

Time, $t = 20$

Energy at, $t = 0$ s

$s = 50$ J.

Energy at, $t = 20$ s

$s = 10$ J

Therefore, $$20 = 50e^{-\frac{b20}{m}}$$

$$\frac{2}{5} = e^{-\frac{b20}{m}}$$

$$\frac{b}{m} = \frac{1}{20}\ln 5$$

21. Assume that the block has a mass 40 kg and the spring has a force constant and that has the force constant k = 36,000 N/m. The block is pulled 2.5 cm below the equilibrium position and released from rest. Find the shortest time, it takes to go from equilibrium to the unstretched position.

Ans. For equilibrium,

$$h = mg,$$

$$h = \frac{mg}{k} = \frac{40\text{ kg} \times 9.8\text{ m/s}^2}{36{,}000\text{ N/m}}$$

$$= 1.09\text{ cm}$$

So going from, $x = 0$ to $x = -1.09$ cm,

the reference circle of radius A = 2.50 cm,

this corresponds to a sweep of $\Delta\theta = 90° - \phi$ on the circle,

where, $\cos\phi = \frac{1.09}{2.50}$;

$$\phi = 64.2°.$$

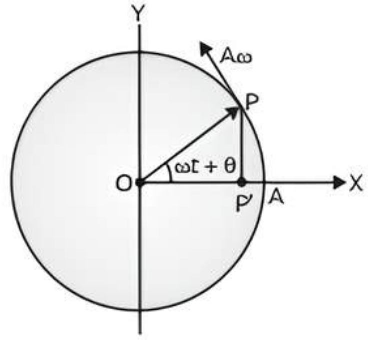
Then, $\Delta\theta = 90° - 64.2° = 25.8°$

and $\Delta t = \left(\frac{25.8}{360}\right)$

$$T = 0.0717(0.209\text{s})$$

$$= 0.01150\text{ s.}$$

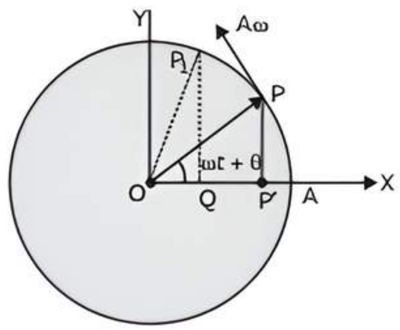
22. In figure, what will be the sign of velocity of the point P' which is the projection of the velocity of the reference particle P. P is moving in a circle of radius R in anti-clockwise direction.



[NCERT Exemplar]

Ans. P' is the foot of the perpendicular of velocity vector of particle P at any time t.

Now particle moves from P to P_1, then its foot shifts from P' to Q, *i.e.*, towards the negative axis. Hence, the sign of θ, motion of P', is negative.



SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

23. A mass M attached to a spring oscillates with a period of 6 sec. If the mass is increased by 3 kg, the period increases by one second. Find the initial mass M assuming that Hooke's law is obeyed.

Ans. We know that,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Where, k = spring constant.

In first case, $$6 = 2\pi\sqrt{\frac{m}{k}} \quad \text{...(i)}$$

In second case,

$$7 = 2\pi\sqrt{\frac{M+3}{k}} \quad \text{...(ii)}$$

Squaring eqn. (i) and (ii) and then dividing (ii) by eq. (i), we have,

$$\frac{49}{36} = \frac{M+3}{M}$$

$$= 1 + \frac{3}{M}$$

On solving we get,
initial mass, M = 8.307 kg.

24. A small object is placed on top of a block of mass, m = 20 kg, that is hung from one end of a vertical spring of force constant k = 2000 N/m and whose other end is attached to the ceiling. The block is pulled down a distance A below the equilibrium position and released from rest. What is the maximum value of A for which the small object remains in contact with the block throughout the subsequent SHM?

Ans. Contact will be lost only if the downward acceleration of the block momentarily exceeds the acceleration of gravity g. When that occurs, the object will not be able to follow the motion of the block. The block's maximum downward acceleration occurs at the highest point in the motion, which corresponds to the amplitude A of motion and is the same as the distance of release below equilibrium. At this highest point, the acceleration has a magnitude

$$a = \left(\frac{k}{m}\right)A$$

Since $a \le g$,

$$a = \left(\frac{k}{m}\right)A = g$$

Gives the maximum Amplitude, A.

$$9.8 \text{ m/s}^2 = \left[\frac{2000 \text{ N/m}}{20\text{kg}}\right]A$$

or $$A = 0.098 \text{ m} = 9.8 \text{ cm}$$

25. The length of a second's pendulum on the surface of the earth is 1 m. What will be the length of a second's pendulum on the moon? [NCERT Exemplar]

Ans. A pendulum of time period (T) of 2 sec is called the second pendulum.

$$T_e = 2\pi\sqrt{\frac{l_e}{g_e}}.$$

$$T_e^2 = 4\pi\frac{l_e}{g_e}$$

$$T_m = 2\pi\sqrt{\frac{l_m}{g_m}}$$

$$T_m^2 = 4\pi^2\frac{l_m \times 6}{g_e}$$

For second pendulum,

$$T_e = T_m$$
$$= 2 \text{ sec}$$

$$\frac{T_m^2}{T_e^2} = \frac{\frac{4\pi^2 6l_m}{g_e}}{4\pi^2\frac{l_e}{g_e}}$$

or $$\frac{(2)^2}{(2)^2} = \frac{6l_m}{l_e}.$$

$$l_e = 1\text{m}$$

$$\frac{1}{1} = \frac{6l_m}{1\text{m}}.$$

$$l_m = \frac{1}{6}\text{m}$$

26. A particle executes S.H.M. period of 4 seconds. After what time of its passing through the mean position will the energy be half kinetic and half potential? [Diksha]

Ans. $$\omega = \frac{2\pi}{8} \text{ rads}^{-1}$$

Let 't' be the time after which the energy will be half kinetic and half potential. Since, motion starts from mean position,

$$y = A \sin \omega t$$

$$\frac{dy}{dt} = v$$
$$= A \cos \omega t \omega$$

$$\text{K.E.} = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m A^2 \omega^2 \cos^2 \omega t$$

This must be half of the total energy.

$$\frac{1}{2}mA^2\omega^2\cos^2\omega t = \frac{1}{2}\cdot\frac{1}{2}mA^2\omega^2$$

$$\cos \omega t = \frac{1}{\sqrt{2}}.$$

$$\omega t = \frac{\pi}{4}.$$

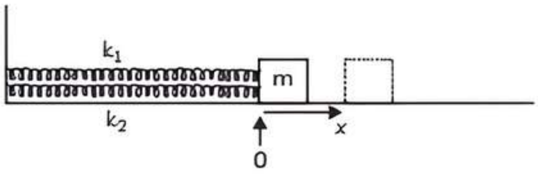
$$t = \frac{\pi}{4}.\omega$$

$$\omega = \left(\frac{\pi}{4}\right)\left(\frac{4}{\pi}\right) = 1 \text{ s}$$

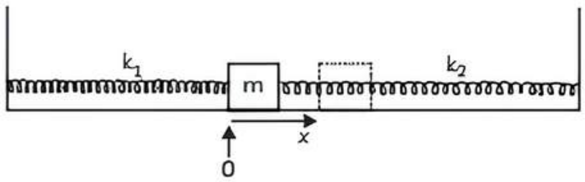
LONG ANSWER Type Questions (LA)

[4 & 5 marks]

27. For the given figure, shown (a) and (b), the solid block represents the equilibrium position and the dashed-line block represents its position after a displacement x from equilibrium. For each case, find the equivalent force constant of a single spring that would have the same effect on the block, thus showing that the block undergoes SHM, find the period of SHM for each case.



(a)



(b)

Ans. For figure (a), in stretching to position x from equilibrium, the restoring force to the left due to both springs is,

$$F = -k_1x - k_2x = -(k_1 + k_2)x.$$

The equivalent single spring would give a force.

Thus, $F = -kx$

$$k = k_1 + k_2$$

and $$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

For figure (b), here the two springs are pulling in opposite directions and just balance at the equilibrium position. If the block is pulled a distance x to the right of equilibrium, then spring one will increase its pull to the left by $k_1 x$, while spring two will decrease its pull to the right by $k_2 x$. Therefore, the net force at position x is $k_1x + k_2x$ to the left. Then, the equivalent single spring will again have a force constant.

$$k = k_1 + k_2$$

and $$T = \sqrt{\frac{m}{k_1 + k_2}}$$

28. One end of a U-tube containing mercury is connected to a suction pump and the other end is connected to the atmosphere. A small pressure difference is maintained between the two columns. Show that when the suction pump is removed, the liquid in the U-tube executes S.H.M. [Delhi Gov. QB 2022]

Ans. Area of a cross-section of the U-tube = A

Density of the mercury column = ρ

Acceleration due to gravity = g

Restoring force, F = Weight of the mercury column of a certain height

$$F = -(\text{Volume} \times \text{density} \times g)$$

$$F = -(A \times 2h \times \rho \times g)$$

$$= -2A\rho gh$$

$$= -k \times \text{Displacement in one of the arms } (h)$$

Where,

$2h$ is the height of the mercury column in the two arms

k is a constant.

$$k = -\frac{F}{h} = 2A\rho g$$

Time Period, $$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2A\rho g}}$$

Where,

m is the mass of the mercury column

Let 1 be the length of the total mercury in the U-tube.

Mass of mercury,

$$m = \text{volume of mercury} \times \text{Density of mercury}$$

$$= Al\rho$$

$$\therefore \quad 2\pi\sqrt{\frac{Al\rho}{2A\rho g}} = 2\pi\sqrt{\frac{l}{2g}}$$

NUMERICAL Type Questions

29. When in mean position, the kinetic energy of a particle performing S.H.M. is 20 J. If the amplitude of the oscillations is 30 cm and the particle's mass is 6.15 kg. How do you calculate the time period of oscillations? (3m)

Ans. The kinetic energy of the particle executing SHM is given by,

$$K.E. = \frac{1}{2}m\omega^2(A^2 - y^2)$$

Given : $K.E. = 20$ J

$$A = 30 \times 10^{-2}$$

$$m = 6.15 \text{ kg}$$

$$y = 0 \text{ m}$$

Also, the time period is given by,

$$T = \frac{2\pi}{\omega}$$

Using the given values in the time period equation, we get,

$$20 = \frac{1}{2} \times 6.15 \times \omega^2 (30 \times 10^{-2})^2$$

Or $$\omega^2 = 0.01 \times 10^{-4}$$

$$\omega = 1 \times 10^{-3} \text{ rad/s}$$

Using, $\omega = 1 \times 10^{-3}$ rad/s in time period equation, we get,

$$T = \frac{2\pi}{1 \times 10^{-3}}$$

$$T = 2000\pi$$

30. **A wire attached to the centre of a circular disc with a mass of 15 kg suspends it. The wire is twisted and released by rotating the disc. The period of torsion oscillation is determined to be 2 seconds. The disc has a radius of 12 cm. determine the wire's torsion spring constant. (The torsion spring constant is defined by the relation $J = -\alpha\theta$, where J denotes the restoring couple and θ the angle of twist.)** **(3m)**

Ans. Mass of the circular disc,

$$m = 15 \text{ kg}$$

Radius of the disc,

$$r = 12 \text{ cm} = 0.12 \text{ m}$$

the torsion oscillations of the disc has a time period,

$$T = 2 \text{ s}$$

moment of inertia of the disc is:

$$I = \frac{1}{2} mr^2$$

$$= \frac{1}{2} \times 15 \times (0.12)^2$$

$$= 0.108 \text{ kg/m}^2$$

Time period,

$$T = 2\pi \frac{I}{\alpha}$$

where, α is the torsional constant.

$$\alpha = 4\pi^2 \frac{I}{T^2}$$

$$= 4 \times \pi^2 \times \frac{0.108}{(2)^2}$$

$$= 1.064 \text{ Nm/rad}$$

Hence, the torsional spring constant of the wire is 1.064 Nm rad^{-1}.

31. **The springs with spring factors k, $2k$ and k are connected in parallel to a mass m. Find the new time period if mass = 0.15 kg m and k = 4 N/m.** **(3m)**

Ans.

$$k' = k_1 + k_2 + k_3$$

$$= k + 2k + k = 4k$$

$$= 4 \times 4, (k = 4 \text{ N/m})$$

$$= 16 \text{ N/m}$$

Time period,

$$T = 2\pi \sqrt{\frac{m}{k'}}$$

$$T = 2\pi \sqrt{\frac{m}{4k}}$$

$$T = 2\pi \sqrt{\frac{0.15}{4 \times 16}}$$

$$T_s = 0.0380 \text{ s}$$